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THE POLYNOMIALS' BOX AND THE TRADITIONAL METHOD: TWO DIDACTIC ALTERNATIVES IN THE TEACHING OF ADDITION AND SUBTRACTION OF POLYNOMIALS

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Abstract

Objective. The purpose of this study was to compare the level of learning achieved by eighth-graders of the Educational Institution (EI) Maria Cano (Medellin-Antioquia) in the topic of addition and subtraction of polynomials when this topic is taught with the traditional method and using the Polynomials' Box didactic tool.

Methods. Four different eighth grade groups are taken and distributed into two groups. Two of the groups are taught using the Polynomials' Box didactic tool, which is based on the Meaningful Learning cognitive theory proposed by David Ausubel. In the remaining groups, the traditional methodology was applied, which is based on explain-exemplify-exercise (hereinafter, EEE). Finally, the results were qualitatively and quantitatively compared.

Results

• Group average grades did not vary from historical averages.
• The difference between averages was relatively small.
• Although grades did not show improvement, the dispersion significantly decreased.

Conclusions. The use of didactic material in classrooms does not guarantee that students will reach a learning goal, but it positively influences relevant aspects in the teaching-learning process, such as: organization, interest and motivation for knowledge, active participation, and interaction, among others.

Key Words: Meaningful Learning, Polynomial's Box, addition and subtraction of polynomials.

1. **Introduction**

Didactics being "*the science of education that studies and intervenes in the teaching-learning process, with the aim of achieving the student's intellectual training"* (Mallat, 2011, p.5), it demands from professionals the production and/or usage of tools to facilitate the teaching-learning process. A considerable number of teachers have taken the process a step further by documenting their work, which is why, currently, there is variety in texts that describe experiences used to teach. Based on the work of Vinholi (2011), who followed the Meaningful Learning theory developed by David Ausubel to teach concepts of botany in schools in Brazil, the authors have implemented methodological strategies within the classroom using the Polynomials' Box didactic material. There are some records in the field of mathematics didactics, such as Urbano (2011) and Valbuena *et al*. (2015), who worked teaching mathematics in schools by means of a computer.

Universities that offer teaching training careers (particularly in the area of mathematics), and even those that do not, have classrooms-workshop spaces specialized in the creation and experimentation of didactic materials to teach different mathematical concepts, to the extent that material could be developed for each topic; this fact added to the learning dynamics of contemporary students, have lead the use of didactic tools to become a need in order to teach mathematics in a classroom.

The Polynomials' Box is an example of material created to teach mathematics. It was created by Masters' Degree Oscar Fernando Soto, a teacher at Universidad de Nariño, with the purpose of providing teachers in the region with a didactic alternative to teach basic operations with polynomials, as well as to teach the factorization process. This material has been applied in certain educational contexts, with a record of the experiences. Soto, Gomez and Mosquera (2005) introduce the academic community with this didactic material, which they called: Polynomial's Box, submitting an article with the same name that was published in issue 13 of Revista de la Escuela Regional de Matematicas (ERM). Soto, Lozano and Naranjo (2009) used this material to teach basic operations with polynomials to people with hearing impairment. Villarroel (2014) adapted this material to generate a didactic proposal to teach the factorization process in addition to basic operations with polynomials, joining the theorical fundamentals of the Polynomials' Box proposal with David Ausubel's Meaningful Learning cognitive theory; eventually, this work turned into his Masters' Degree thesis.

Any mention of Meaningful Learning necessarily must refer to the work of Marco Moreira, who in an international meeting on meaningful learning exposed that people learn based on what they already know, a principle that originated (in its time) Ausubel's proposal. Additionally, he incorporated the idea to abandon the narrative in the classroom, which he called Critical Meaningful Learning, Moreira (1997).

The intention of this work is to take two out of four eight grade groups in the EI Maria de los Angeles Cano Marquez (Medellin-Antioquia) and apply the proposal resulting from an association of the Polynomials' Box and of Ausubel's Critical Meaningful Learning theory to teach addition and subtraction of polynomials; while the two remaining eight grade groups work with the conventional method EEE (explain-exemplify-exercise) used to teach these concepts. But mostly, it is to analyze the results obtained after applying both methodologies, from a quantitative and qualitative perspective. The first, applying some descriptive statistics concepts, and the second establishing a chart comparing the characteristics of aspects considered critical to develop a class. Along with those figures, the didactic material is disclosed and its main characteristics are explained, providing teachers with the information required at the moment of selecting a didactic tool to be used in the classroom.

2. **Methodology**

The grades studied in the work were 8.1, 8.2, 8.3 and 8.4 of the EI Maria Cano in the municipality of Medellin (Antioquia). The selection of the grades to which the methodologies were applied was random, and they were differentiated as follows: EEE methodology: 8.2 and 8.4 (hereinafter, EEE groups), and the Polynomials' Box methodology: 8.1 and 8.3 (hereinafter, CP groups).

2.1. Design of Didactic Material

The learning guidelines for the CP groups were selected in this stage, which were designed by one of the authors of this document as part of a graduation thesis for a Masters' Degree in Science Teaching. It must be highlighted that each student in the CP groups provided the raw material to build the Polynomial Boxes, which were built by a group of eight-graders with the supervision of the teacher.

2.2. Previous Organization

According to Ausubel's theory, it is indispensable that the person who is learning has the necessary knowledge to learn the new information, Ausubel termed this pre-concepts or pre-requisites as subsumers (Moreira, 1997). Therefore, it was a priority for the study to have the four groups work on and understand the following subsumers: basic operations with real numbers, exponentiation and reduction of like terms; the technical part was emphasized, since these operations are applied in basic operations with polynomials.

The work with the Polynomials' Box implied the need to dedicate two additional sessions alongside the CP groups, in order to explain how the material worked, it was critical for all students to master the representation processes of polynomials, reduction of like terms, among others.

2.3. Description of Methodologies

2.3.1. EEE Method

Traditionally (and based on specifically mathematical reasons), the operations with polynomials were developed similarly as basic operations with real numbers, in the following order: addition and then subtraction (understood as a sum with an opposite additive).

Addition

Explain: it was explained in the board that, in order to add polynomials, a new polynomial is created by adding the polynomials with the "+" sign, and then the like terms, if any, are reduced.

Exemplify: this stage of the method works with several examples, trying to address as many as possible. The following was proposed among them:

Being and . Determine:

Also, an alternative process is indicated, locating one of the polynomials to be added in a line, and the other one to be added in the line below, guaranteeing that each term of the second to be added is placed below its like term in the first one, proceeding to simplify the like terms.

Being and . Determine:

Exercise: exercises of a higher level of complexity are proposed. First, addition of monomials, addition of binomials and finally, addition of polynomials.

Subtraction

Explain: it was explained that if there are two polynomials, and , two new polynomials may be determined: and . Also, that the polynomial is interpreted as the sum

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between the polynomial and its opposite polynomial, so in order to build an opposite polynomial, the signs of the terms in the initial polynomial have to be changed.

Another relevant aspect is that the relation between and is explained, since a difference consists of precisely the opposite of another, meaning or .

Exemplify: considering the and polynomials, determine.

As mentioned, since polynomials need to be added, this requires creating the opposite polynomial of

Now, the addition takes place, the students select their method of choice. The following is a reference to the process we have called alternative.

Exercise: as with addition, exercises that range from least to highest complexity are proposed.

2.3.2. Polynomials' Box Method

The following is an analysis of the experience of applying the learning guidelines aimed at fostering the learning process of addition and subtraction of polynomials, based on David Ausubel's Meaningful Learning theory.

Construction of the Material

The Polynomials' Box is a didactic tool that may be used to teach different concepts of school mathematics, which could lead to the assumption that it is only manufactured by specialists; however, the students provided the materials to conduct this research, and most importantly, they built their own Polynomials' Boxes, with the supervision of the area's teacher.

Theoretical-mathematical Fundamentals of the Polynomials' Box

The Polynomials' Box is a tool with mathematical fundamentals in all of its elements: pieces, board and functioning.

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Pieces

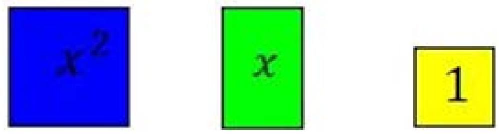

Figure 2.1: pieces of the Polynomial's Box.

There are three different kinds of pieces, illustrated in Figure 2.1: large squares, rectangles and small squares. The large square (blue) has a defined distance, assigned as the measurement of its sides, with the area of the square being . For the rectangle (green), that distance is one of the dimensions and a unit distance (1) is defined for the other, resulting in a rectangle with and dimensions, with the area being . Finally, a small square (yellow) is built from the sides corresponding to the unit, with the area being . These pieces are related because the dimensions of the rectangle match the dimensions of the other two exactly, allowing the pieces to be precisely joined by their corresponding sides, the resulting figure is illustrated in Figure 2.2.

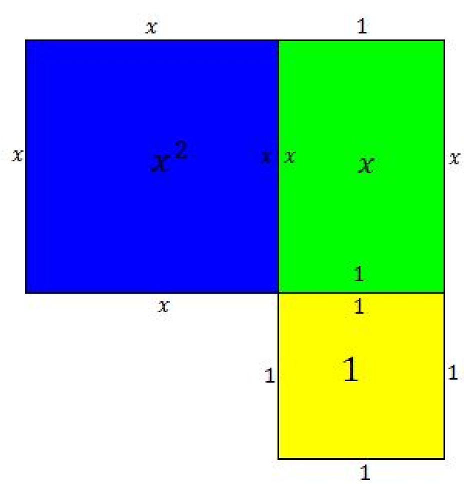

Figure 1.2: relation between the dimensions of the pieces.

Board

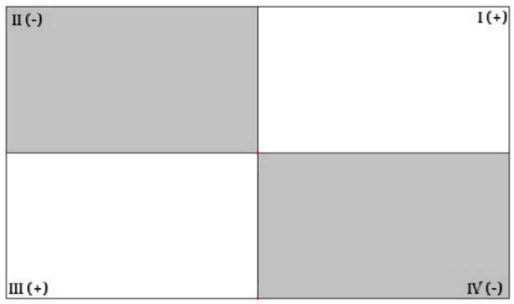

Figure 2.3: board.

The board is a rectangular region that emulates the Cartesian plane, it is divided in four sub-regions with two perpendicular straight lines, joining the midpoints of the sides. As it can be seen, each sub-region has a name that resembles the plane's quadrants and its signs. The top right quadrant is the first one and it will have a positive sign, the top left will be the second quadrant and it will have a negative sign, the bottom left is the third quadrant and it will be positive, and the bottom right quadrant is the third and it will be negative. The allocation of the signs is based on the same foundation as the signs in the plane's quadrants, thus, the board of the Polynomial's Box also uses coordinated axes. The intersection of the lines will be the origin, the segment between the origin and the board's top midpoint to the right will represent positive values, and between the origin and the board's top midpoint to the left will represent negative values. Likewise, the segment from the origin to the board's bottom midpoint to the right will represent positive values, and between the board's bottom midpoint to the left will represent negative values.

Relation between Pieces-Board

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To place the pieces in the board it is important to understand two main aspects. FIRST: the area of all of the pieces must be totally contained in one of the plane's regions because placing a piece in a region will determine its sign. SECOND: if one or two side of a piece are placed on the axes, the pieces would get the sign of the axis.

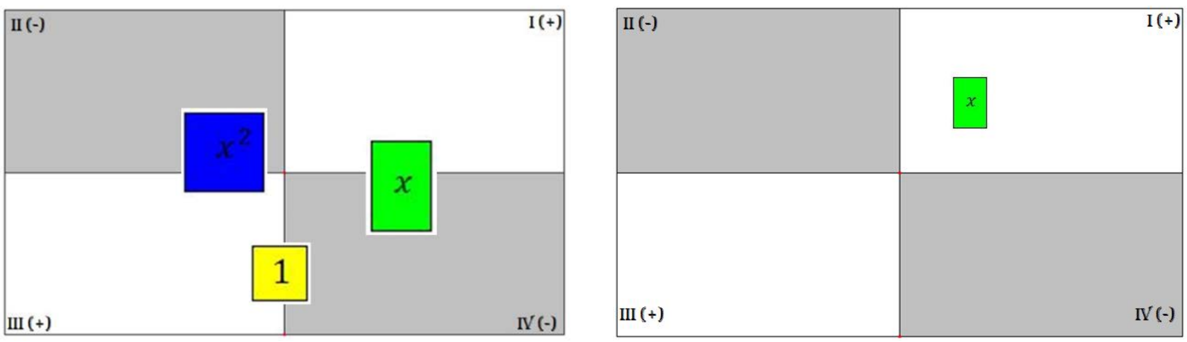

Figure 2.4: correct placement of the pieces.

Figure 2.4 (to the right) shows the correct way of placing a piece on the board, with the whole piece contained in the first quadrant. On the other hand, Figure 2.4 (to the left) shows three pieces that are incorrectly placed because the pieces are on both quadrants.

Writing Polynomials in the Board to Add and Subtract

Polynomials that are with , are used, meaning second degree polynomials in a single variable with integer coefficients. Writing polynomials depends on the operation intended to be solved in the board.

Quadrant signs allow representing the coefficient signs, while the number of pieces placed on the board represent the coefficient's absolute value, therefore there are different ways of representing the polynomial in the board. One of them would be placing two pieces in the second quadrant; three pieces in the first quadrant and a piece in the fourth quadrant. Figure 2.5 shows two different placements of the pieces representing the polynomial.

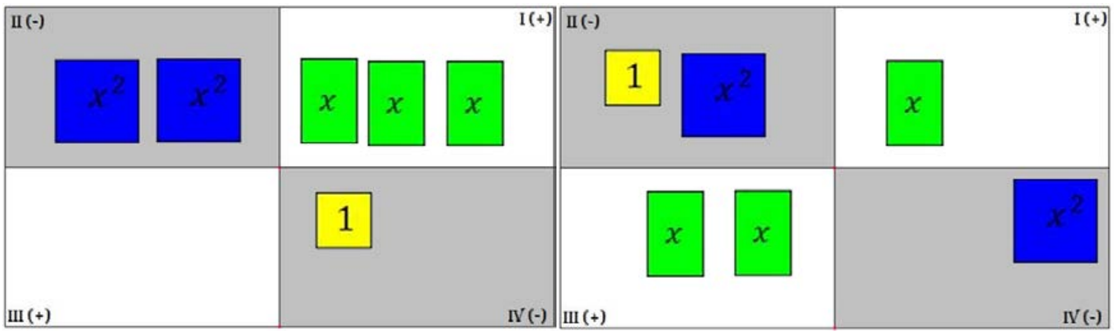

Figure 2.5: representation of addition.

Execution of Basic Operations with the Polynomials' Box

Zeros

A fundamental concept to be used is called **zero**, which in mathematical terms refers to the reversibility of addition of real numbers. The Polynomials' Box allows using the pieces to represent such an essential property of addition in a way that is easy to understand, two pieces of the same algebraic weight that are placed in quadrants with opposite signs are opposite, with its addition resulting in zero; it is easy to represent zero in the board in multiple ways.

Addition

Since addition is a binary operation, two polynomials must be represented in the board, being critical for pieces corresponding to each polynomial to be identified. It is recommended that the first and fourth quadrant be used to represent one of the addends, and the second and third quarter be used to represent the other addend, thus preventing each polynomial's pieces to get confused.

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After the two addend polynomials are represented in the board, the reversibility of addition of real numbers is applied, i.e., looking for the highest amount possible of zeros. Depending on the sign of each polynomials' term, pieces of the same type may be placed in quadrants with the same sign, in that case, those result in zeros, therefore the pieces must be added, implying pieces to be translated to a same quadrant. Finally, and after removing the pieces that result in zeros, the polynomial that is represented in the board is read. Said polynomial is the addition of both initial polynomials.

Being and . Figure 2.6 shows the representation of polynomials in the board. see how the pieces of polynomials are placed in quadrants with the same sign, respectively, this means they do not result in zero and must be added, to do so, they must simply be put together. If the 1 pieces result in zero, it implies a 1 piece from each polynomial must be removed from the board, then the polynomial that is represented in the board is read, being the addition between and . Figure 2.6 shows the addition of polynomials represented in the board, being and .

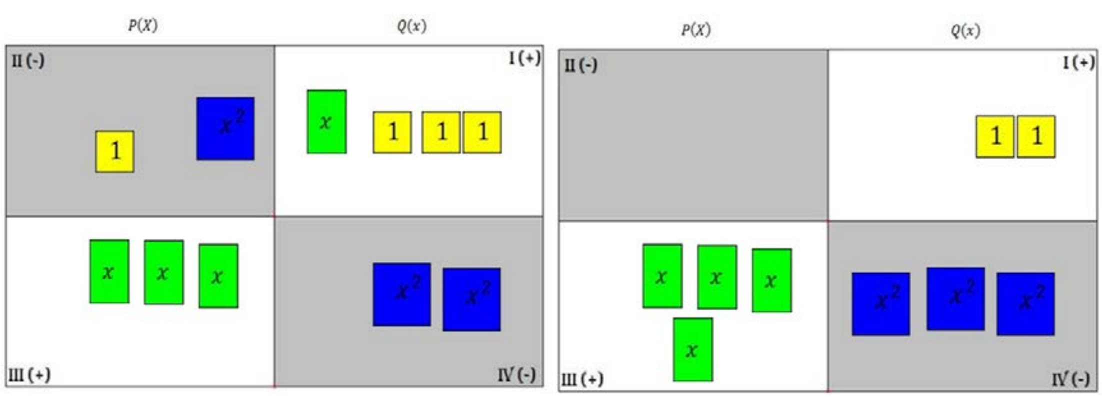

Figure 2.6: location of the addends.

Subtraction

Reversibility can be applied to the addition operation for the polynomials set, therefore, for any given polynomial, there is another polynomial and both result in a zero polynomial when added, in that case said polynomials are mutually opposite.

This essential property allows a polynomial to be built based on its opposite. In practice, this is done changing the signs of the polynomial's coefficients, thus,

Representation of the Opposite Polynomial in the Board

As previously mentioned, subtraction is an addition in which the addends are a polynomial and the opposite of another polynomial; in that sense, the same that was defined for polynomials addition will apply for polynomials subtraction. The differential element between both operations is that operation , understood as , meaning, the addition of the polynomial and its opposite polynomial.

To be represented in the board, each polynomial must be placed in two quadrants (As established for addition), but once the subtrahend polynomial is placed its opposite is formed by changing the pieces to another quadrant, for instance, if the pieces of the subtrahend polynomial are placed in quadrants I and IV, the construction of its opposite polynomial results from moving the pieces from quadrant I to quadrant IV, and vice versa. Mathematically, this means changing the sign of the terms of the polynomial, as shown in Figure 2.7.

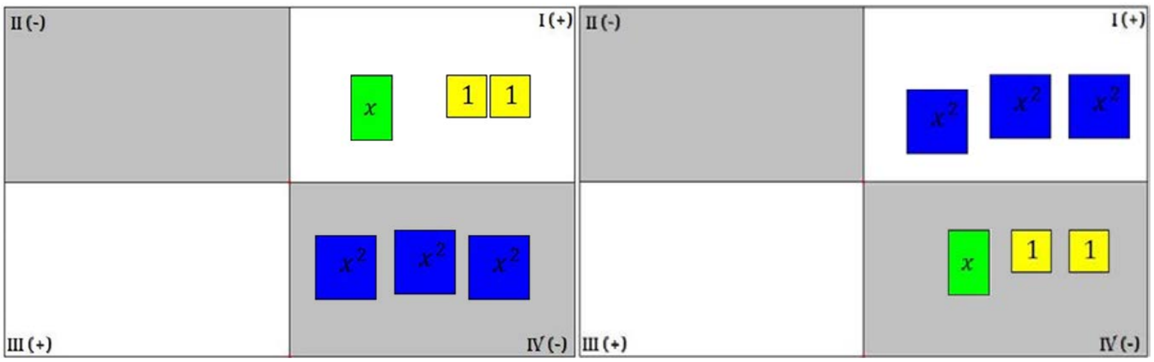

Figure 2.7: P polynomial and opposite P polynomial.

Process to subtract

Once the subtrahend and addend polynomials are placed, and after the opposite polynomial of the subtrahend polynomial has been built, the process is almost done because all that is left is to identify the largest number of zeros between the pieces of the subtrahend and addend polynomials; just as in the addition, the like term pieces placed in quadrants with the same sign must be moved. Finally, the polynomial in the board will be the difference intended by the operation.

Figure 2.8 shows an illustration of the identification of zeros between the pieces of and , which are removed from the board; when the pieces with the same sign are moved between quadrants, the polynomial is represented in the board, as follows:

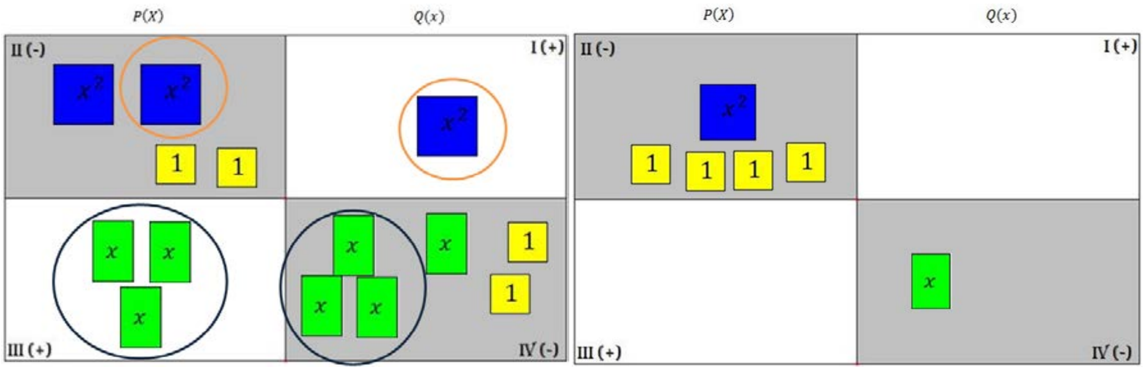

Figure 2.82: identification of zeros and result of the subtraction.

2.4. Polynomials' Box and Meaningful Learning

Relation: Material-Cognitive Structure

Ausubel's theory of Meaningful Learning suggests, among other, that the material presented to a person that is learning must be potentially meaningful, this means there needs to be a previous relation between the material and a concept already acquired in the student's cognitive structure (Moreira, 1997). The characteristics and properties of the elements that compose the Polynomials' Box are highly binding with the knowledge that an eight-grader has incorporated into his/her cognitive structure since these concepts have been studied since elementary.

The different pieces of the Polynomials' Box correspond to squares and rectangles, geometrical shapes that every student can identify from an early age. The area of each piece represents the term's literal part. However, the pieces acquire a sign when placed in the board, if placed in quadrants I or III, the pieces will have a positive sign, likewise, if placed in quadrants II and IV, the pieces will have a negative sign. The terms' coefficients are represented in the number of units of each piece. It must be said that the Polynomials' Box allows the representation of terms with integers, with the objective of involving an abstraction process that leads to adding or subtracting polynomials (with any coefficient) without using the tool.

It is evident that the components of the material (pieces and board) are elements with which the students are familiar, their attributes are also concepts that have been studied from early ages of the students' school cycle, consequently this satisfies the relation between material and cognitive structure proposed by Ausubel.

Subsumers

As previously mentioned, Ausubel explains that before introducing new knowledge, a stage to work with the required knowledge (subsumers) is needed (Moreira, 1997). For the case of addition and subtraction of polynomials using the Polynomials' Box, the subsumers are as follows: Cartesian plane, and perimeter and area of squares and rectangles. Ausubel's theory proposes that for each new topic, subsumers must be worked on as abstractly as possible, aiming at students reaching the maximum appropriation level to use and apply them seamlessly when needed (Moreira, 1997). This is why, before working with the Polynomials' Box, two sessions with the aforementioned subsumers were conducted.

Subsumption

Subsumption is the process by which new information is anchored to a concept that has been previously established in the student's cognitive structure in order to incorporate it (Moreira, 1997). In the work with the Polynomials' Box, the concepts denominated by Ausubel as anchors are: the concept of area of squares and rectangles, the addition of amounts with the same sign, different sign and opposite signs.

Accordingly, it must be mentioned that subsumption is the ultimate objective of a person that intends to teach, but in each case, it will be realized in different ways. The experience with the Polynomials' Box was not an exception, it was carried out in different stages, which comprehensively result in the students learning the concepts of addition and subtraction of polynomials, which translated into an abstraction and incorporation to the students' cognitive structure of the process of adding and subtracting any type of polynomials without the need of using or representing them in the Polynomials' Box.

Classwork

As mentioned before, the CP groups had two extra sessions to apply the learning guidelines of the Polynomials' Box. The intention was to provide an approach to the material, for them to manipulate it and even play with it, since play is at the core of the Polynomials' Box, (Soto *et al*., 2005). Afterwards, the functioning, characteristics and fundamentals of each element of the material was explained; in this stage, it was crucial not to overlook any details, the more binding the elements and concepts were for the students meant higher possibilities for them to anchor the new knowledge.

When the groups were familiar with how the Polynomials' Box worked (understood how to correctly place the pieces in different quadrants, represented any term in different ways and, overall, correctly represented and read any polynomial allowed by the tool in the board) the guideline for the addition and subtraction of polynomials was applied. This guideline explains the process needed to add and subtract polynomials, and also proposes an activity to add and subtract using the Polynomials' Box; initially, this activity aims at developing operational ability followed by inducing the student to abstract the method to formally add and subtract polynomials, all of which is based on the practice, questions and answers.

3. **Analysis of the Experience**

The main motivation that drove the planning and execution of this work (which was never an interest to undermine a specific working methodology) was to quantitively and qualitatively identify the benefits offered by each one at the moment of choosing a teaching methodology for addition and subtraction of polynomials, because the selected methodology must positively respond to several pedagogical, didactic, organizational and logistical variables. Hence, it is necessary to conduct studies that allow the characterization of all possible choices that a teacher has when teaching, which will lead to the selection of the one that adjusts the most to the needs of a particular group, being aware of the differences between the groups.

3.1. Characterization of the Groups

The groups selected to apply this working methodology are made up by males and females between the ages of 12 and 17, an adequate age range for an eight-grader in a formal EI. None of the participants had been diagnosed with learning difficulties or special education needs, nor with physical impairment that hinders the learning process.

EEE Groups

Groups 8.2 and 8.4, to which the EEE methodology was applied, were constituted by 43 and 46 students, respectively, males and females, each.

CP Groups

Groups 8.1 and 8.3 were constituted by 43 and 40 students, respectively, males and females. No repeaters were included. A major differentiating factor was that the students in the groups were easily concentrated and willing to work. Group 8.3 had a determining absence factor that negatively affected the learning process, some students had high levels of absence. On the other hand, Group 8.1 was affected by high levels of absence but only on two, maybe three, students.

3.2. Quantitative Analysis

The quantitative analysis of the efficiency achieved by each method considered two different tests.

Solution of exercises: it was proposed to verify the level achieved by each student. It consisted on 10 pairs of polynomials that had to be added and subtracted. The proposed polynomials got increasingly more complex: the first polynomials had an integer coefficient and a single letter in the literal part; the second polynomials had fraction coefficients and two or more letters in the literal part; the final polynomials had terms with fraction coefficients, two or more letters in the literal part and literal exponents. The grading scale was the Institution's official scale, ranging between 0 to 5 points. Each exercise was assigned an equal numerical value of 0.5 per exercise.

Presentation: to establish if the students had incorporated the addition and subtraction of polynomials to their cognitive structure, each student had the assignment to present the process of addition and subtraction of a pair of polynomials in a private interview with the teacher, providing argumentations for each step. This process was considered correct if the student managed to properly explain the process, and incorrect in any other case.

The following tables and corresponding illustrations include the results obtained by each group in each activity.

Solution of Exercises

Table 2.1: number of exercises that were correctly solved per group.

Range	8.1	8.2	8.3	8.4
0 to 5	9	14	10	14
6 to 7	23	12	21	20
8 to 9	8	12	4	8
10	3	5	5	4
Total	43	43	40	46

Figure 2.9: bar diagram of exercises that were correctly solved per group.

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Table 2.2. Averages and standard deviations per group and methodology.

	8.1	8.2	8.3	8.4	CP	EEE
Average	3.2	3.0	3.0	2.9	3.1	2.9
Standard	1.1	1.6	1.2	1.4	1.2	1.5

Table 2.2 shows that the averages per group and working methodology do not have a significant dispersion, implying that this is not the aspect in which the effectiveness of the alternative methodology focuses. But the standard deviations of each group and methodology lead to the conclusion that the alternative methodology accomplishes all students to come closer to the mean (grade-wise); on the other hand, the traditional methodology generates higher grade dispersion with students getting very high and very low grades.

Presentation

Table 2.3. Defenses per group.

Assessment	8.1	8.2	8.3	8.4
Correct	25	14	16	22
Incorrect	18	29	24	24
Total	43	43	40	46

Figure 2.10: bar diagram of presentation results per group.

3.3. **Qualitative Analysis**

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Instead of being a compilation of arguments by different authors, the following analysis arises from a pedagogical and didactic reflection that took place after the experience had been completed. Teachers' years of service, variety of taught groups, continuous and reiterated teaching of eight grade algebra, voluntary or involuntary knowledge of the students' behavior patterns, allow them to identify the characteristics that are needed in a working methodology of each group they work with. It is an undeniable fact that if a specific methodology has a positive effect on a group, it is impossible to guarantee the same effect with another group. In that regard, a need arises in terms of analyzing the characteristics of experience that are excluded from the quantitative analysis, there are many relevant aspects in a classroom that are not perceived by numbers.

The intention of this analysis is not to undermine any methodology, on the contrary, it is to try to find aspects that support and positively contribute when selecting a working methodology to teach addition and subtraction of polynomials, notwithstanding an adaptations and applications to other topics. This is why a parallel between both methodologies has been drafted, aimed at showing the strengths of each methodologies in aspects we consider basic and fundamental at the moment of choosing a teaching method.

Table 2.4. Comparative Table Between Methodologies.

CATEGORY	EEE GROUP	CP GROUP
TIME	It is easily manipulated, depending on the intended degree of complexity it may be previously predetermined and discriminated.	Time is predetermined, but there is a need for additional sessions to explain how the Polynomials' Box works.

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MOTIVATION	Very low levels are observed, with the exception of students that enjoy math.	The fact of working with materials different from pencil and paper generates expectations. Also, since the material is based on a traditional game (puzzle), it generates feelings of joy.
ORGANIZATION	This method makes it easier to control the organization of the groups.	The class' initial order is highly altered, especially in the first sessions.
TEACHER-STUDENT INTERACTION	Interaction is much higher in the explanation stage since this is where doubts are solved. There are interaction peaks when the student faces the first exercises.	In certain moments, teacher-student interaction becomes frequent, especially in when the foundation work begins and in the last part of the guideline (where the student faces the abstraction process); it also occurs when facing the exercises to verify if the goal of the process was met.
RESOURCES	There was an effort to provide a package containing the exercises to each student with the objective of optimizing time.	Wood board, cardboard, guillotine, scissors, vinyl, markers. Copies of the learning guidelines, copies of the verification exercises.
SATISFACTION	From the point of view of learning, the students expressed satisfaction because they learned and interiorized the	From the point of view of effective learning, students expressed that it is really complicated to establish a correlation

	technique to add and subtract polynomials. From the point of view of enjoyment and pleasure deriving from working with this methodology, most students stated that the process was not of their liking (as expected) since it seems "scripted", it followed a structure of explanation and drills to solidify the process. Students claimed, "it is very boring".	between the fundamentals of the Polynomials' Box and different types of polynomials to add or subtract, meaning that the abstraction process is very complicated. Students manifest that the experience of working with the Polynomials' Box was nice, they never imagined that topics as abstract as this one could be tackled with a tool that is based on play and that is made up of colored squares and triangles.
STUDENT-STUDENT INTERACTION	Given that the material was provided to each student, peer interaction was really minimum; moreover, the teacher allowed all questions arising from the explanation stage to be formulated to him in person, in order to be more precise in its solution.	Despite the fact that the material was built for each student, as expected, there was a lot of interaction based on the excitement of being playing instead of learning. Play is a social activity and given the fact that the tool is based on play, it was logical for interaction to be high.

4. **Conclusions**

1. The didactic material acts as a facilitator of interaction and exchange of knowledge in the classroom. Compared with the traditional methodology, the Polynomials' Box widely exceeds in teacher-student and student-student interaction levels.
2. Working with the Polynomials' Box (following Ausubel's criteria), enables the formulation of questions by students because the new information is associated with concepts that are already incorporated to their cognitive structure; this implies that the student does not need to have high-level abstraction processes to be able to formulate questions, since there is an association to prior knowledge.
3. The Polynomials' Box reduces dispersion of students' grades, which means that most of them would acquire the same appropriation levels of the new information. With the traditional method, grade average is very similar to that obtained with the Polynomials' Box, but the dispersion index is more elevated, which means that some students get very high grades, but in contrast, some get very low grades; it is possible to infer that while some learn to add and subtract polynomials very well, others have difficulties to learn those concepts.
4. The abstraction process is highly complex because it requires detailed planning to lead students to successfully accomplish it. Although abstraction is not being taught, the formulation of questions allows mental processes (typical of mathematical thinking) to abstract properties, processes, among others, of tangible objects being used in the classroom, thus converging in learning the concepts that are taught.

5. Teaching with the traditional methodology may achieve very high levels of appropriation of the new information, since the process is being developed abstractedly with objects that are typical to the topic, but it requires the student to have a high level of own subsumers for each topic, since those are responsible for linking the new information to the one in their cognitive structure.

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