PANORAMA
ISSN: 1909-7433
ISSN: 2145-308X
ednorman@poligran.edu.co
Politécnico Grancolombiano
Colombia

ADDITION BETWEEN FRACTIONS AS PART OF A WHOLE USING PLAY WITH A GAME OF A3 STRIPS

Martinez Villalba, Maria del Carmen; Meza Salgado, Armando; Agudelo Marin, Yaneth Milena

ADDITION BETWEEN FRACTIONS AS PART OF A WHOLE USING PLAY WITH A GAME OF A3 STRIPS
PANORAMA, vol. 13, núm. 25, 2019
Politécnico Grancolombiano, Colombia
Disponible en: https://www.redalyc.org/articulo.oa?id=343963314015
DOI: https://doi.org/10.15765/pnrm.v13i25.1265

Los autores/as que publiquen en esta revista aceptan las siguientes condiciones: Los autores/as conservan los derechos de autor y ceden a la revista el derecho de la primera publicación, con el trabajo registrado con Creative Commons: Reconocimiento - No Comercial -Sin Obra Derivada, que permite a terceros utilizar lo publicado siempre que mencionen la autoría del trabajo y a la primera publicación en esta revista. Los autores/as pueden realizar otros acuerdos contractuales independientes y adicionales para la distribución no exclusiva de la versión del artículo publicado en esta revista (p. ej., incluirlo en un repositorio institucional o publicarlo en un libro) siempre que indiquen claramente que el trabajo se publicó por primera vez en esta revista. Se permite y recomienda a los autores/as a publicar su trabajo en Internet (por ejemplo en páginas institucionales o personales) antes y durante el proceso de revisión y publicación, ya que puede conducir a intercambios productivos y a una mayor y más rápida difusión del trabajo publicado.

Artículos de investigación científica y tecnológica

ADDITION BETWEEN FRACTIONS AS PART OF A WHOLE USING PLAY WITH A GAME OF A3 STRIPS

Adición entre fracciones como parte de un todo utilizando el juego con regletas a3

Maria del Carmen Martinez Villalba marvi_201@hotmail.com
Universidad Autonoma de Manizales., Colombia
Armando Meza Salgado armameza@hotmail.com
Universidad Autonoma de Manizales, Colombia
Yaneth Milena Agudelo Marin
magudelomarin@autonoma.edu.co
Universidad Autonoma de Manizales, Colombia

PANORAMA, vol. 13, núm. 25, 2019

Politécnico Grancolombiano, Colombia

Recepción: 09 Marzo 2017
Aprobación: 09 Julio 2019

DOI: https://doi.org/10.15765/pnrm.v13i25.1265

Redalyc: https://www.redalyc.org/articulo.oa?id=343963314015

Abstract: The objective of this research was to analyze changes in the understanding of the process of adding fractions as part of a whole with the use of A. strips with 4^{th} grade students in a State Educational Institution in Cordoba (Colombia). The study follows a qualitative methodology detailing the process related to the acquisition of knowledge regarding the sum of fractions as part of a whole. Emphasis is placed on play as a methodological strategy, applying the game of A. strips to guide students so that they can become competent to interpret, represent and apply the sum of fractions as part of a whole. A series of instruments were introduced in its development, which allowed to diagnose the obstacles shown by students in terms of the subject of the study. In this investigation a didactic unit related to logical mathematical thinking was planned and executed, based on a model followed by three concrete step-by-step moments, these three moments consist of activities that allow the student to construct his/her own knowledge as he/she is carrying out activities that involve conceptual evolution. An analysis of the results is shown, registering before, during and after of the intervention of the didactic unit.

Keywords: Fraction, part-whole relationships, addition between fractions, game, A. strips.

Resumen: El objetivo de esta investigación fue analizar cambios en la comprensión del proceso de la adición entre fracciones como parte de un todo, a partir de la utilización de las regletas A3, con los estudiantes de grado 4º en una institución educativa estatal de Córdoba (Colombia). El estudio presenta una metodología cualitativa en la que se detalla el proceso relacionado con la adquisición del conocimiento referente a la suma de fracciones como parte de un todo. Se hace énfasis en el aspecto del juego como estrategia metodológica, basado en la aplicación de las regletas A3; mediante estas se orientó a los estudiantes para pudieran ser competentes para interpretar, representar y aplicar la suma de fracciones como parte de un todo. En el desarrollo de esta se presentó una serie de instrumentos que permitieron diagnosticar los obstáculos que los estudiantes registraban frente a la temática del estudio. En esta investigación se planeó y ejecutó una unidad didáctica relacionada con el pensamiento matemático lógico, fundamentada en un modelo seguido por tres momentos concretos paso a paso; estos tres momentos constan de actividades que admiten que sea el estudiante quien construya su propio conocimiento en la medida en que va realizando las actividades donde evoluciona conceptualmente. Se muestra un análisis de los resultados teniendo en cuenta el antes, durante y después de la intervención de la unidad didáctica.

Palabras clave: Fracción, relación parte todo, adición entre fracciones, juego, regletas A3.

INTRODUCTION

The life of a child is characterized by play, it is an activity that consumes a "large" portion of his/her life, it can be said that it is their main interest up to 12 or 13 years of age, approximately. Likewise, play becomes a process that enables children to discover external reality, to progressively transform their ideas regarding their connection with the world (Lopez, 1989, p.21).

Childhood is the foundation for this research proposal; it is precisely the stage of the fourth graders at Nuestra Señora de la Candelaria Educational Institution, they are going through or experimenting a need to know and interact with the world around them, along with the playful stimulation provided by the school, aimed not at numeric content but applied as a methodological resource to learn mathematics.

Playful and/or recreational activities mixed together with the didactic-mathematic process provides children with different possibilities of models and imagination that can adjust to their capacities and their way of understanding the context prior to attaining full appropriation. In this process, the teacher acts as facilitator of a methodology that allows students to know and understand reality surrounding them, and to quantitatively and qualitatively relate information and problems that will further their mental and academic development.

This research proposal has been conceived based on a constructivist outlook of learning.

Nowadays, it is inappropriate to refer to constructivism with a unified approach because there are diverse conceptions, Ausubel, Driver and Vygotsky agree that: "children spontaneously acquire their own concepts about natural phenomena of the external world based on their own development and without the direct influence of adults" (Claret, 1996, p.2).

Therefore, educators are responsible for the pedagogic, strategic or scientific innovation required to attain the results of children's work and to turn the pedagogical activity into an enjoyable moment, filled with learning and collaboration, in order to awaken their interest in mathematics as an opportunity for growth and social development.

Taking the aforementioned into consideration, it is indispensable to apply a didactic strategy based on play to aid the logical understanding of addition between fractions as part of a whole, as well as to comprehend, interpret, solve, propose and calculate problems related to this operation with fourth graders of the elementary level at the studied Educational Institution in Planeta Rica, Cordoba.

Theoretical Framework

Considering the purposes, categories and subcategories of this research, the following are the theoretical foundations:

Concept of Fraction

A review of the concept of fraction led to different approaches; this work will apply the ideas of:

Freundenthal (1983, p.10) who established that "fractions are the phenomenological resource of rational numbers, a fountain that never runs dry. It is the word that introduces rational numbers and it is connected with breaking: fracture".

Understanding the division of a unit, meaning going from the concept of natural numbers to fraction numbers, implies having worked on the concept of unit, its division in congruent parts with the status of the number (taking fraction units into consideration: (½, 1/3, ¼, 1/5...), without losing the notion of unit, as well as extending the meanings of the concept of fraction numbers in any given situation, i.e., knowing how to contextualize them.

Knowing how to contextualize the system of fraction numbers will lead students to interpreting fractions in different contexts. Authors such as Kieren (1993) and others, state that partitions and distributions in equal parts must have a privileged place in selecting basic competences required to learn fractions. Brousseau (1981, 1986), another renowned author, has insisted on the differentiation between fraction, measurement and lineal operation in the construction, for students be able to observe mathematical models that aim to generate situations from physical problems that might have certain results (rational). On the other hand, discrete or continuous contexts are relevant for different ways to solve understanding partitions and distributions (Steffe & Olive, 1990), (Streefland, 1991). Moreover, Douady (1986) favors interactions between mathematical and physical frameworks that drive the necessary invariants to conceptualize rational numbers.

Didactic studies by Gallardo and Rojano (1988), Vasco (1994), Rojano (1994), Ohlsson (1988), Mancera (1992), Obando (1999), Freudenthal (1994-b), Martinez C and Lascano M (1999), Llinares. S and Sanchez. M (1998), Carretero (1986, 1987 and 1989) focus on the proposal of general lines for the construction of rational numbers within the school context, and some of them have allowed the study of variables from the cognitive perspective.

On the other hand, different analyses and studies conducted by pedagogues on teaching mathematics from different points of view, have shown that students conceptualize the numeric system of natural numbers through operations and relating them to their learning experiences in school. Authors such as Brissiaud (1989) and Kamii (1984, 1985) concur that comprehending a number is much more than learning a numeric sequence and learning reading and writing of numerals, they consider that it is, above all, the process of appropriating a sign system as a cultural tool in different contexts in which children have to solve problems concerning the comparison of extension of amounts in several sets.

Systems of Representation Used in Part-Whole Relationships

Linares and Sanchez (1988b) propose to firstly take on concrete objects by translating them to oral and written representations, taking into account that it is necessary to start by introducing the meaning as activities with manipulative material are conducted and as translations are achieved using written and oral representations that use symbols and words, it is also intended for this results process to work in the opposite direction, meaning that with a written and oral representation of fractions in connection with part-whole meaning, students can translate it to a concrete representation using the material they are given to manipulate.

Bruner (1984) differentiated three key types used by people to represent their mental models and reality. First, the inactive system, such as sensory and motor processes of physical experiences. Second, the iconic system used to represent things through an image or space sketch regardless of the action. And third, the symbolic system used to represent a thing with an arbitrary symbol whose shape seems unconnected to the thing being represented.

Addition of Fractions

In terms of addition of fractions, important ideas such as those proposed by Gairin (2003, p.248) can be taken into account:

> "Concepts such as addition and subtraction of positive fractions are associated to the aggregation or disaggregation of amounts of the same magnitude. The corresponding calculus algorithms are justified by the need to measure both quantities with the same subunit and thus, by the need to operate with equivalent fractions".

Thus, the process is understood as a combination of one or more fractions of an equivalent number (called addition) which is represented by a + symbol.

Solving Problems and Fractions

In the teaching-learning process of mathematics, students attempt to consolidate their skills and dexterities to solve problems. The NCMT (2000, p.55) proposes:

> "Solving problems is a comprehensive part of learning mathematics and it is precisely why it should not be an isolated part of this discipline's program. Solving problems is not just a learning objective of mathematics, but also one of the main ways to achieve it."

It is important to realize what solving a problem in mathematics implies, there are different views on this matter, Santos (1997) says that the problem is related to the relativity of a person trying to solve a situation, what may be a problem for someone may be an activity likely to be solved for others. The situations that students solve need to be connected with their experiences, context and scientific or work competences. It is of great importance for the teacher to develop the dexterity and skill to solve problems because this entails a school environment that motivates them to explore, encourages them to research and socialize their results with one and other.

The article *How to Teach Children to Solve Mathematics Problems* proposes ideas based on evolutionary psychology and states that children under twelve years of age need to have physical contact with the objects introduced in the situation of the problem, and to manipulate them in order to better understand it, since they lack the ability of effective abstract thinking, therefore it is key to introduce children the problems in a concrete way, turning the abstract into concrete (Cardelli, 2012).

It is important to consider that while solving problems, many factors, processes or strategies need to come together, one of them being the metacognitive process that the student undertakes when faced with a situation that demands higher performance. According to the ideas presented by the Research Group on Science Learning (physics department, Universidad de Alcala), which consists of Juan Miguel Campanario Larguero, Jose Cuerva Moreno, Aida Moya Librero and Jose C. Otero Gutierrez (p. 40): metacognition is recurrent in mastering the skill of solving problems in sciences. Solving problems is a significant source of difficulties for students and some authors validate overall failure. For this task, researchers took into consideration the works of authors such as (Gil, Carrascosa, Furio and Martinez-Torregrosa, 1991); (Gil, Martinez-Torreposa and Senent, 1988). On the other hand, and following mechanic effectiveness, students rarely analyze the validity of the problem's solutions, so that solutions that are numerically absurd are accepted as valid without difficulty (Campanario, 1995). Moreover, this group considered the work of Swanson (1990), who studied the strategies used by subjects with high and low levels of academic aptitude and metacognition while solving problems, results indicate that subjects with high levels of academic aptitude and metacognition apply a richer set of strategies; individuals with high metacognition level solved problems better than individuals with low metacognition, but subjects with high metacognition level and low academic aptitude performed significantly better than subjects with high academic aptitude but low metacognitive level. This seems to indicate that a high metacognitive level may compensate deficiencies in academic aptitude while solving problems (Campanario, Cuerva & Moya, 1998, p.40).

This process applied diverse strategies, students were probed making use of different types of metacognitive questions: aimed at processes, requiring precision and accuracy (descriptive), open to encourage divergent thinking, to select alternative strategies, to lead to reasoning, prove hypothesis or insist on the process, overall motivating, and with questions that encourage reflection and control impulsiveness (How did you do it? Which strategies did you use to solve it? Are there other options? How else could it have been done? Is there another answer or solution? Why did you do it like that and not differently? Why did you write or say this? What would happen if you would use another figure instead of this one? Which steps did you take to complete your task?).

The process of solving problems cannot ignore the importance of cognitive sciences' teaching-learning process, which involves graphical representation to symbolize and examine problems with the purpose of understanding formulations and planning maps to find the solution. Some of these evinced skills and dexterities are: charting a scheme, finding a similar solution, clearly simplifying the situation. "Successive acquisition of increasingly complex logical structures which underlie different tasks and situations the subject can solve as he/she makes developmental progress" (Piaget, 1979, p.102).

It is said that people show incomparable progress stages and daily practices. Analyzing mind methods, such as attention, perception, language, memory, analysis and problem solving, categories and concepts, cognitive development, representations, awareness and learning. The main goal is to understand how these processes take place in people, trying to justify what happens in the interior world.

Play in Pedagogy

Various concepts to define play exist under many approaches, from ancient times to our days, from the Greeks' classical education of Homer's time, when it was considered an "elegant distraction of gentlemen" (Marrou, 2004, p.28). The 18^{th} century's modern thinking consolidated a theory of play's nature, contributing and rescuing its importance in culture and education, enabling liberty and vitality and making it an indispensable element in the development of human beings. Play also provides joy, pleasure, satisfaction, and can make a child create, dream, travel or shift between fiction and reality (Triana, 2013).

Experts in psychology and pedagogy affirm that play in children is a major mental and physical activity that leads to student progress in pleasant and comprehensive ways. Play is the way in which children manifest, it is a form of language used by children to let their personalities show: in childhood's early years, play must be encouraged with different functional games to help them grasp location in space and time, psychomotor coordination, sensory and perceptive progress (Crespillo, 2010, p.14).

It has been said that play is a fun practice that involves pleasure, enjoyment, cognitive, social and emotional progress necessary to understand the game and its different expressions in the educational environment; but especially, play is understood as an excuse to make progress in creative thinking stages from the valuation of converging and diverging structures (Romero, 2013).

Didactic Unit

A didactic unit is a work unit of variable duration, it organizes a set of teaching-learning activities and -at its highest concentration level- responds to all of the curriculum's elements: what, how, when to teach and evaluate. Moreover, the didactic unit is understood as a programming unit that incorporates the intervention and participation of all the elements involved in the teaching-learning process, it also has an implicit methodological coherence and a determined time frame (Antunez ,1992).

The way in which a didactic unit is produced and executed in connection with logical mathematical thinking is based on a model that follows three concrete step-by-step moments, these are made up of activities that enable students to progressively build their own knowledge as they conceptually evolve (Sanchez, Castaño and Tamayo, 2015).

..Strips

A. strips are a game of mathematical application created by a group of teachers of basic secondary education in 2000; its name comes from the initials of the creators (Armando Meza, Armando Quintero, Antonio Barrios). The strips are used to teach concepts of fractions as part of a whole. There are 30 strips in total (Meza & Barrios, 2010).

Methodology

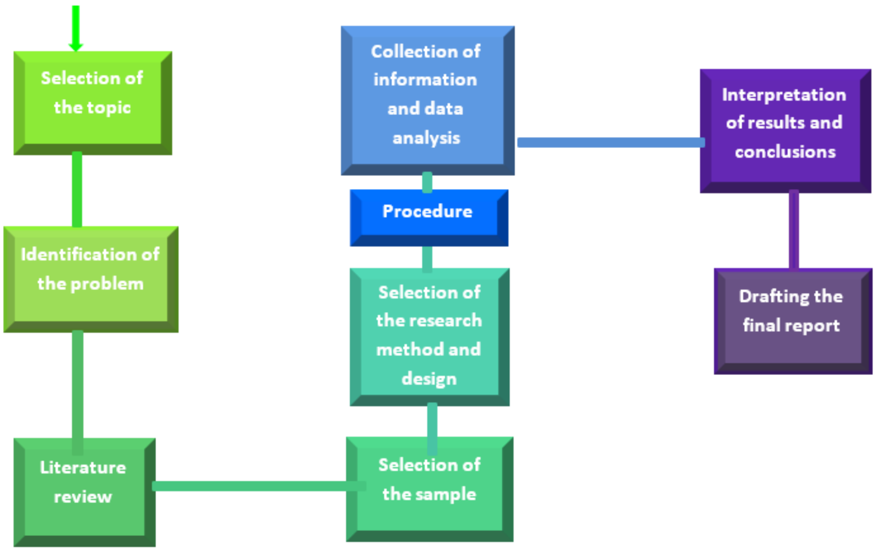
Regarding the term "qualitative research", Strauss and Corbin (2002) affirm that it is any type of research that does not resort to statistical procedures or any other means of qualification. Qualitative methods can be used to explore substantive areas of which little is known but in which new knowledge is pursued (Stern, 1980). Thus, qualitative methodology is considered pertinent for this research with the aim of analyzing changes in understanding the process of addition between fractions as part of a whole with the use of A. strips.

Qualitative research is indispensable and agrees with the type of research developed because it provides three main components to consider: first, it feeds from data obtained through different information sources, in this case surveys were applied and observations, documents and applicable records were produced in each stage of the research. Second, applied procedures, which aid organization and interpretation of data obtained, such as: conceptualizing and reducing data, building term categories of properties and dimensions and corresponding them with coherent systematization. Third, written reports, which serve the purposes of socialization of results to the academic and scientific community in journals, lectures and congresses.

In synthesis, the qualitative research applied in this work allowed theories substantiated on obtained data to generate knowledge, increase comprehension and metacognition while providing a significant guideline to learn what to do with knowledge.

The framework of the research proposal encompassed the following stages:

Scheme of Stages of the Research Process



Graph 1
Stages of the research process
Source. Compiled by the authors

Concerning the characterization of the work unit, the data collection instruments (questionnaires, A. strip sets) were applied to forty fourth graders in Nuestra Señora de la Candelaria Educational Institution, aged 9 and 10. The sample obtained corresponds to 25% of the students, who were randomly selected, the instruments were applied to the entire fourth grade in order to avoid students from feeling excluded from the process, but at the moment of systematizing the data, only the randomly selected percentage was considered.

The categories to analyze in this research were given by the production and execution of a didactic unit in relation to logical mathematical thinking, which is substantiated in a model followed by three concrete step-by-step moments, these are made up of activities that enable students to progressively build their own knowledge as they conceptually evolve.

The first moment is called location, and it detects students' difficulties, which are evident based on their previous ideas. The second moment is called dislocation, it is the application of didactic strategies based on the analysis deriving from the first moment, it displays different interpretations of the concept, involves authentic problems and questions of metacognitive type aimed at students' self-regulation in relation to previously identified obstacles. The third moment is called refocus and it tackles the subject matter of the study with more elaborate problems that involve prior components (obstacles, use of different languages, metacognitive self-regulation questions) (Sanchez et al., 2015b).

Results

The results presented are the interpretation and descriptive analysis that allowed the triangulation to be conducted, keeping the categories and subcategories in mind.

Subsequently, throughout the design and application of the A. strips game with the students who participated in this research, their evident enjoyment was reflected in their gesticulations and verbal expressions which denoted great pleasure, constant interaction was also manifest, they undertook the activity with the joy of play, and, without them knowing it, sustained meaningful learning as per (Triana, 2013b).

During the pre-tests, ten students showed difficulties related to the problem statement and resolution, seven of them also had difficulties solving addition between fractions and four had the aforementioned plus one connected with the interpretation of fractions as part of a whole. Collecting the data provided by the population that is the subject of this study in the different moments of play with the .. strips, the findings in students are as follows:

(1-JM) who initially had difficulties in distinguishing fraction terms and functions, overcomes it by playing with the A. strips. After the activity, the student expresses it has been useful because he/she managed to conceptualize fractions and addition as part of a whole, wrote problems and solved them, although he/she still evinces difficulties while writing the statement. His/her text showed predominance of graphic representations.

(2- AM) the student successfully overcame difficulties presented, which were related to **addition, statement and solving problems.** He/she produced problem statements and solved them; therefore the application of game of A. strips was useful and this experience allowed the student the possibility to turn abstract into concrete. This was observed in the graphic representation he/she provided as an answer.

(3- AY) following the activity with the A. strips (manipulation of strips – concrete) this student shows it has been fruitful because he/she overcame difficulties related to the **distinction of each term of the fraction, as well as their function, addition, statement and solving problems**. He/she was able to understand problem statements and to present solutions, the progress made with the A. strips is noticeable. As evidence of the foregoing, he/she presented a problem, which he/she was able to solve in abstract with a graphic representation.

(4-JJ) in this case, although the student's problem statement still has room for improvement, the activity with the game of A. strips was clearly useful for him because he was able to get over his difficulties, which were related to **problem statement and solving problems**, he wrote problems and presented solutions; it can be said that (4-JJ) made improvements with the game of A. strips in realizing the problem's answer, namely an addition, placing the strips together and finding the strips that would represent the answer and placing them on top of the others (concrete), and also making an abstract representation in arithmetic terms.

(5-MM) appropriates signs in different contexts to solve the proposed problem, has writing mistakes although the solution proposed is accurate. The game of A. strips largely improved the difficulties presented, which were related to addition, **problem statement and solving problems**.

(6-FG) shows improvement in difficulties related to **problem statement and solving problems**, he/she wrote problems and presented solutions in a graphical way, justified answers orally when socializing his/her production.

(7LM) initially, the student had difficulties **distinguishing the terms of fractions, as well as their function, addition, statement and solving problems**. After the application of the game of A. strips, he/she correctly distinguished fraction's terms, addition problem statements and solutions and explains the solution to other classmates, his/her progress was evident in the graphic and abstract representations.

(8JP) had difficulties **distinguishing each term of fractions, as well as their function, addition, problem statement and solving problems.** This situation was corrected with the game of A. strips, there is an evident improvement because the student manages to formulate problems, although it is necessary to continue reinforcing argumentation and writing, his/her work was graphic and abstract.

(9-SA) initially, the student had difficulties regarding **addition, problem statement and solving problems**, after manipulating the game of A. strips, the student was able to state and solve problems in a graphic and abstract way.

(10-JM) had difficulties in terms of **problem statement and solving problems.** The situation was overcome because the student presented problems and its solutions both graphically and in abstract.

The prior situations reprise contributions by: Freudenthal (1983c), Streefland (1991,1993b), Peralta (1994) and Goffrec (2000). Accordingly, it is necessary to favor settings that acknowledge the development of reflexive knowledge to grant students the chance to build relations and symbolic representations through their own practice, leading them to the realization, interpretation, discussion and representation of procedure processes for problems regarding fractions and their description in concrete and symbolic levels. Students mentally redo their daily experiences in a context of interaction and playful conditions while learning fractions.

This requires the mental process of abstraction. "The process of abstraction is extremely complex since it needs detailed planning in order for students to successfully achieve it. Abstraction is not taught, but through a process of asking questions it is possible to induce mental processes that are typical of mathematical thinking aimed at abstracting traits, processes and others of tangible objects used in the classroom, which finally converge in learning the concepts that are intended to be taught". (Villarroel, et all, 2017 pag.32)

Evolutionary psychology suggests that children under twelve years of age need to have physical contact with the objects introduced in the situation of the problem, and to manipulate them in order to better understand it, since they lack the ability of effective abstract thinking, therefore it is key to introduce children the problems in a concrete way, turning the abstract into concrete (Cardelli, 2012).

When children manipulate the A. strips they explore and find their own equivalences in surfaces, although they sometimes argue that they “take the same space” as is they were referring to volume; manipulating and confronting the strips led them to build a sense of equivalence, concurring with the idea exposed by Cardelli.

The concepts of addition and subtraction of positive fractions are associated to aggregation or disaggregation of amounts of equal magnitude. Corresponding calculus algorithms are justified by the need to measure both amounts with the same subunit, thus, by the need of operating with equivalent fractions (Gairin, 2000b).

By establishing equivalences with A. strips, students were able to get different answers to addition between fractions in an agile way.

As psychology and pedagogy experts claim, for children play is a vital mental and physical activity that pleasantly and comprehensively aids student progress. Play is the way in which children manifest themselves, a type of language through which they let their personality surface; throughout the children’s training, play must be encouraged with different functional games to help them grasp location in space and time, psychomotor coordination, sensory and perceptive progress (Crespillo, 2010, p.14b).

The aforementioned was confirmed while applying the didactic unit based on the game of A. strips, children’s manifestations, gestures, words, graphics and texts ratifying their progress in terms of understanding the process of addition between fractions as part of a whole.

The game of A. strips was applied as a methodological strategy that contributes to logical understanding of addition between fractions as part of a whole among fourth graders, it was done as a hidden curriculum because they focus their attention on the game and its results without realizing they are achieving meaningful learning about fractions but expressed in another context: it has been said that play is considered a fun practice that involves pleasure, enjoyment, cognitive, social and emotional progress needed to understand the game and its different expressions in the educational environment; but especially, play is understood as an excuse to make progress in creative thinking stages from the valuation of converging and diverging structures (Romero, 2013b).

DISCUSSION

The implementation of this work proposal with fourth graders has been proven through the analysis of processes that have resulted in advantageous changes in understanding the process of addition between fractions as part of a whole using the game of A. strips. These changes are clear in student processes with different mathematical representations, such as:

Improved student conceptualization regarding addition between fractions as part of a whole (concrete) reflected in the use of the game of A. strips, an initial diagnosis identified weaknesses or difficulties in students' conceptualization, after the application of the strategy, it became clear that students efficiently built a conceptualization of addition between fractions as part of a whole.

Development of student activities allowed evincing the progress in understanding addition between fractions as part of a whole using a graphic representation of the proposed mathematical operation.

Solutions to problems proposed by the teacher and the student implied the application of knowledge about addition between fractions (abstract), this process was achieved after playing a game of A. strips; students became more involved, understanding and reasonable regarding addition between fractions as part of a whole, consequently, an environment of meaningful learning was generated. Progress in understanding the process of addition between fractions as part of a whole was obvious in most students, they tried uncommon alternatives for similar situations that needed to be resolved, thus proving that mathematics can be fun (playful) and can be contextualized using daily activities to better comprehend and analyze real situations.

A study entitled *Teacher Training and Professional Development Focused on Language and Mathematics Teaching in Colombia,* in agreement with findings in the literature review, concludes: "the majority of studies show concern and interest by all actors involved in the educational system to achieve improvement in training for teaching and learning of necessary reading and mathematics skills at regional, national and international level, starting in early childhood's areas of education of language and mathematics". (Gonzalez, 2018 Pag. 15)

Implementing play with the game of A. strips enhances relationships with students because this activity provides a fun approach that leaves aside the apathy that many students feel towards mathematics.

Students fulfilled the development of competences required to understand addition between fractions as part of a whole, which is why this game can be used as a new tool to teach mathematics to fourth graders.

REFERENCES

Antúnez. (1992). Necesidad de la planificación en los centros escolares. Revista Aula de Innovación Educativa, 1, 43-48. Recuperado de http://www.grao.com/revistas/aula/001-la-programacion-didactica--planificacion-y-gestion-analisis-de-necesidades/necesidad-de-la-planificacion-en-los-centros-escolares.

Brissiaud, R. (1989). El Aprendizaje del cálculo más allá de Piaget y de la Teoría de Conjuntos. Madrid, España: Aprendizaje Visor.

Brousseau, G. (1983). Les obstacles épistémologiques et les problèmes en mathématiques. Recherches en Didactique des Mathématiques, 4(2), 165-198.

Bruner, J. (1984). Desarrollo cognitivo y educación. Madrid, España: Ediciones Morata

Campanario, J.M. (1995). Los problemas crecen: a veces los alumnos no se enteran de que no se enteran. Aspectos didácticos de física y química (física), 6, pp. 87-126. Zaragoza: ICE, Universidad de Zaragoza.

Campanario, L., Cuerva M., Moya L. (1998). La metacognición y el aprendizaje de las ciencias. Investigación e innovación en la enseñanza de las ciencias, 1, pp. 36-44.

Cardelli, J. (2012). Articulo como enseñar a los niños a resolver problemas matemáticos. Barcelona, España: Editorial Fontalba

Claret, A. (1996). El constructivismo según Ausubel, Driver y Vygotsky. Revista actualidad educativa, 3(12), 20-31.

Crespillo, E. (2010). El juego como actividad de enseñanza aprendizaje. Gibralfaro. Revista de Creación literaria y Humanidades, 68, 14-20. Recuperado de: https://issuu.com/yuly4/docs/el-juego-como-actividad-de-ense__an_c6ccb2a2132b51

Douady, R. (1986). Acercarse verdadera situación de los números aprendizaje (de 6 a 11 años) de la escuela. Investigación Matemáticas en Educación, 11, pp. 77-110.

Freudenthal, H. (1983). Fenomenología didáctica de las estructuras matemáticas. Dordrecht, Países Bajos: Reidel.

Gairin, J. (2003-2004). Estudiantes para maestros: Reflexiones sobre la instrucción de los números racionales positivos. Contextos Educativos, 6 (7), 235-260. Universidad de Zaragoza. Recuperado de: https://publicaciones.unirioja.es/ojs/index.php/contextos/article/view/538/502

Goffree, F. (2000). Principios y paradigmas de una educación matemática realista. En Gorgorió, Deulofeu y Bishop. (Coords.), Matemática y Educación. Retos y cambios en una perspectiva internacional (pp. 151-158). Barcelona, España: Editorial Grao.

González, A. & Díaz, A. M. (2018), Formación docente y desarrollo profesional situado para la enseñanza del lenguaje y matemáticas en Colombia. Revista Panorama, 12(22), 7 - 17. DOI:http://dx.doi.org/10.15765/pnrm.v12i22.1136

Kamii, C. (1984). El número en la educación preescolar. Madrid, España: Aprendizaje Visor

Kamii, C. (1985). El niño Reinventa la aritmética. Madrid, España: Aprendizaje Visor

Kieren, T. (1993). Rational and fractional numbers: From quotient fields to recursive Understanding. En Llinares, S. (2003) Fracciones, decimales y razón. Desde la relación parte-todo al razonamiento proporcional. Chamorro, C. (Coord.)., Didáctica de las Matemáticas (pp. 187-220). Pearson-Prentice Hall.

Marrou, I. (2004). Historia de la Educación en la Antigüedad. Sao Paulo, Brasil: Herder Editorial.

Meza, A. y Barrios, A. (2010). Propuesta didáctica para la enseñanza de las fracciones. Recuperado de http://funes.uniandes.edu.co/1174/1/674_Propuesta_Didctica_Asocolme2010.pdf

NCTM. (2000). Principios y estándares para la educación matemática. Primera edición en castellano. Sociedad Andaluza de Educación Matemática, Thales. Sevilla

Piaget, J. (1979). Tratado de lógica y conocimiento científico. Naturaleza y métodos de la epistemología. Buenos Aires, Argentina: Paidós.

Romero, V. (2013). El juego infantil y su metodología. Barcelona, España: Altamar.

Sánchez-Castaño, J. A., Castaño-Mejía, O. Y. & Tamayo-Alzate, O. E. (2015). La argumentación metacognitiva en el aula de ciencias. Revista Latinoamericana de Ciencias Sociales, Niñez y Juventud, 13 (2), 1153-1168. Recuperado de http://www.scielo.org.co/pdf/rlcs/v13n2/v13n2a43.pdf

Santos, L.M. (1997). Principios y métodos de la resolución de problemas en el aprendizaje de las matemáticas. Ciudad de México, México: Grupo Editorial Iberoamérica

Stern, P. N. (1980). Grounded theory methodology: Its uses and processes". Image; 12, 20-23

Strauss, A. y Corbin, J. (2002). Bases de la investigación cualitativa. Técnicas y procedimientos para desarrollar la teoría fundamentada. Universidad de Antioquía: Colombia

Streefland, L. (1991). Las fracciones en la educación de la matemática realista. Un paradigma de la investigación del desarrollo. Dordrecht, Países Bajos: Editores académicos Kluwer

Swanson. (1990). Intluence of metacognitive knowledge and aptitude on problem solving. Journal Educational Psychology, 82 (2), 302-314.

Triana, L. (2013). El juego en la pedagogía como lenguaje. Revista Internacional magisterio. (61) 18-22. Recuperado de https://issuu.com/revistamagisterio/docs/revista_internacional_magisterio_n__5d48d07a4e5fb0

Villarroel Solis, J. M., & Romero-Leiton, J. P. (2017), La caja de polinomios y el método tradicional: Dos alternativas didácticas para la enseñanza de la suma y la resta de polinomios, Panorama 11(20), pág. 20-32. DOI: http://dx.doi.org/10.15765/pnrm.v12i22.1136