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# DEVELOPMENT OF MATHEMATICAL COMPETENCES IN GEOMETRIC THINKING THROUGH POLYA'S HEURISTIC METHOD

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## Abstract

This article presents the results of a research aimed at evaluating the effectiveness of Polya's heuristic method (1981) in the development of spatial thinking mathematical competences. This research was developed with a quantitative approach and a quasi-experimental design; a test was used to identify performance on reasoning skills, problem-solving and communication in geometric thinking of two groups of fifth-graders at Villa Cielo Educational Institution, located in the municipality of Monteria (Cordoba-Colombia), before and after an intervention. A didactic strategy was applied in the topic of geometric solids, taking into account Polya's model steps for problem-solving and cooperative work strategy. The results obtained were analyzed by means of statistical student's T-test. It was evidenced that after the intervention, students significantly improved in the performance of competences, corroborating the strategy's effectiveness.

**Keywords:** geometric thinking, problem-solving, Polya's heuristic method, competences.

## Introduction

Teaching and learning geometric thinking, one of mathematics' components, is overall affected since studies conducted reveal that it is usually left for the last academic term and that it is limited to the transmission of formulas and drawings (Gonzalez and Guillen, 2006).

Teaching geometry in elementary education is important because students develop skills to visualize, explore, represent and describe objects in their surroundings, providing them knowledge that is useful in their daily lives. Yet, the Ministry of National Education (MEN, 1998) states in its Basic Competence Standards that: “geometry’s abstractions and rigorousness make it difficult to teach and learn it”.

Research conducted by Gutierrez (1998) found that teaching geometry in elementary education is based on basic knowledge of plane and spatial figures, on learning formulas to calculate areas and volumes (usually developed in the last academic terms).

The results of the PISA standardized assessment in the area of mathematics for the year 2012, rank Colombia in the last position among members of the Organisation for Economic Co-operation and Development (OECD), meaning students are failing to develop the minimum competences required to perform in society.

Moreover, standardized tests conducted in Colombia by the Colombian Institute for the Promotion of Higher Education (ICFES, for its Spanish acronym), show that students fail to achieve satisfactory performance in the area of mathematics; 78% rank in minimum and poor levels, which means that they do not develop mathematical competences to solve problem situations throughout.

The results of the tests in the municipality of Monteria, and specifically of the Villa Cielo Educational Institution, are similar to those obtained at national level, 79% and 91%, respectively, are discriminated in minimum and poor levels, which means that the students fail to solve problem situations of lesser complexity.

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It must be mentioned that, aside from the students' deficient performance, weaknesses exhibited in geometric competences and especially in the geometric component became the objects of study of this research. It is convenient to highlight that researchers such as Baez and Iglesias (2007) and Paredes *et al*. (2007) mention that in most educational establishments, geometry is traditionally taught, based on group work and teacher's lecture as main didactic tool; Institutional Educational Projects are neglected and geometry is taught using paper, pencil, board and marker, hindering the student's chance to develop his/her own creativity in the quest for lasting and effective learning.

Also, Hernandez and Villalba (2001) affirm that if geometry is presented to the student as a final and finished product, he/she is denied the possibility to encourage creativity and to develop skills and competences conducive to meaningful learning. In that regard, the proposed didactic strategy becomes largely relevant in the development of mathematical competences based on Polya's heuristic method (1981).

The general objective of this research was to evaluate the efficacy of Polya's heuristic method (1981) in the development of the mathematical competences of geometric thinking in fifth-graders at the Villa Cielo Educational Institution.

Taking into account the general objective of this research, the following presents the inception, definition and characteristics of problem-solving, its steps, geometric thinking skills and cooperative work.

## Conceptual Framework

### Problem-Solving in Geometric Thinking Teaching

Studies on teaching mathematics show problem-solving as a strategy or tool through which students build their own knowledge. Therefore, it is suggested that geometry is taught based on problem-solving involving geometric relations and concepts (Peña, 2008).

For Polya (1981), problem-solving is a method that allows the student to use different heuristics to solve a problem. Heuristics comes from the Greek word *heuriskein*, which means to find or discover. A heuristic method is a set of strategies applied to solve problems and of decision rules used by problem solvers, and it is based on previous experience.

According to Peralta (2000), the heuristic method is an activity that contributes to the training of active students, builders of their own language. For instance, geometry is not the study of shapes, but of properties that remain invariant in terms of certain practical transformations.

In his book "How to Solve It", Polya describes heuristics as the art of problem-solving. Heuristics tries to understand the method that leads to problem-solving, particularly of mental operations that are useful to this process (Polya, 1965, p. 102). His method is synthesized in four steps:

**Table 1.**

***Steps of Polya's heuristic method***

| STEPS OF THE MODEL BY GEORGE POLYA | | |
|---|---|---|
| **1.** | **Understand the problem** | Reads the problem in detail, expresses it with his/her own words, identifies the data of the problem discriminating necessary information, elaborates diagrams of graphs. |
| **2.** | **Devise a plan** | Proposes different strategies to solve the problem, including: looking for similarities with other problems, enunciating the problem in a different way, looks for different heuristics for the solution. |
| **3.** | **Carry out the plan** | Implements selected strategies, reviews the correct aspects of the strategy to solve the situation and uses new ones, given the case. |
| **4.** | **Look back** | Provides reasons for the solution to the proposed situation.<br>Compares different solution strategies.<br>Analyses strategies to solve other problems. |

## Mathematical Competences

The OECD's PISA study (2015), defines this competence as:

> The capacity of an individual to identify and understand the role of mathematics in the world, to undertake adequately justified reasoning and to use and get involved in mathematics in a way that satisfies the needs of the individual's life as a constructive, committed and reflexive citizen (p 12).

On the other hand, Tobon (2006), considers competences to be complex performance processes suitable in a specific context, with responsibility. Moreover, MEN's Basic Competence Standards (2003) affirm that: "being mathematically competent is connected with knowing what, how, when and why to do something", this is a sign that relates competences to doing and to

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understanding what and how something is done, and to the attitude and disposition to conduct the activity.

In the process of relating mathematical concepts with context situations, mathematical competences play a key role as knowledge generators axes. Said competences are classified by MEN (2006) in five processes, namely: 1. Communication, 2. Reasoning, 3. Modeling, 4. Problem formulation, treatment and solving, 5. Procedure formulation, comparison and exercise. For the purposes of this research, these competences are defined according to ICFES' (2007) classification.

**Communication and Representation**: according to MEN (2006), this competence is:

> Acquiring and mastering languages characteristic of mathematics must be a deliberate and careful process that enables and encourages regular and explicit discussion of situations, senses, concepts and symbolizations; to become aware of the connections among them and foster collective work, in which students share the meaning of words, phrases, graphs and symbols, appreciate the need for collective and universal agreements, and value the efficiency, efficacy and economy of mathematical languages (p. 54).

**Problem Modeling, Proposal and Solving:** this competence allows understanding problematic situations, developing, applying and justifying diverse strategies to solve proposed situations (ICFES, 2013).

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**Reasoning and Argumentation:** reasoning is the set of skills, knowledge and attitudes regarding the explanation of the processes assumed to solve a problem (Tobon, 2007).

**Cooperative Work**

For Ferreiro and Calderon (2001) cooperative knowledge is an innovative educational model that offers a different way of organizing school education in different levels: a school as a whole, in a way that it is an institutional organization model; and a classroom, being a teaching and learning organization; it can also be considered as a learning method or technique.

Pujolas (2009), defines cooperative learning as:

> The didactic use of small teams of students, generally heterogeneous in performance and capacity, although it may be more homogeneous, using the activity's structure to assure equal participation and enhance simultaneous interaction among them (p. 231).

## Methodology

Analyzing the research's characteristics and taking into account the fact that group allocation was not random, a quasi-experimental design was chosen in order to evaluate the impact of change treatments and/or processes in situations in which objects of study have not been randomly allocated (Arnau, 1995).

Therefore, two groups (control and experimental) were conformed. The experimental group was intervened with didactic strategies mediated by Polya's heuristic method through the cooperative work methodology, with the purpose of developing mathematical competences; they took a pretest and a posttest (see Annex 1).

The study's population were fifth-graders at the Villa Cielo Educational Institution, which has three groups in the afternoon session, a total of 102 students. The selection of the experimental and control groups was random, as follows: $5^{th}$ 1 (control) and $5^{th}$ 3 (experimental).

The following variables were identified, taking this research's general objective into consideration:

The independent variable in this research is the intervention of a didactic strategy based on Polya's heuristic method (1981) on geometric thinking in the study unit of geometric solids, using the cooperative work methodology, aimed at developing mathematical competences (reasoning, communication and problem-solving). Each stage of the intervention proposed different situations related and unrelated to mathematics.

The following were taken into consideration to produce didactic guidelines regarding the geometric solids study unit: the Basic Competence Standards in Mathematics (MEN, 2006, p. 82), Basic Learning Rights (MEN, 2015), Mathematics Reference Matrix (MEN, 2015), all of which are related to geometric thinking (Annex 2).

**Table 2.**

***Summary of the Intervention's Work Sessions***

| Learning Unit | Geometric Solids | |
|---|---|---|
| Session's Name | Learning Purpose | Competence Indicators |
| Prints of geometric solids | Discovering the faces and the characteristics of geometric solids' prints | **Communication**: describing tridimensional shapes according to shape and characteristics<br>**Reasoning**: comparing and classifying tridimensional objects and bidimensional shapes according to components<br>**Problem-solving**: using geometric relations and properties to solve problems |
| Comparing objects in the surroundings and classifying geometric solids | Interpreting, comparing and justifying the properties of bidimensional and tridimensional shapes | **Communication:**<br>• Differentiating objects' measurable attributes<br>• Identifying objects' attributes that can be measured, length, surface, space it occupies.<br>**Reasoning:**<br>• Comparing and classifying tridimensional objects and bidimensional shapes according to components and properties<br>**Problem-solving:**<br>• Using geometric relations and properties to solve problems<br>• Using representations |
| Visual perception of objects | Graphically representing the different bidimensional views of a tridimensional shape | **Communication**: Differentiating objects' measurable attributes<br>**Reasoning:**<br>• Representing tridimensional objects based on bidimensional representations<br>**Problem-solving:**<br>• Using geometric relations and properties to solve problems |
| Solids in our surroundings | | **Communication**: describing tridimensional shapes according to shape and characteristics<br>**Reasoning:** identifying solids' properties and characteristics<br>**Problem-solving:** solving problems that require identifying patterns and regularities using geometric representations |
| The art of building solids with common materials | Building and manipulating spatial shapes and graphic | **Communication**<br>• Describing procedures to build shapes and figures based on the measures given<br>**Reasoning** |

| | | |
|---|---|---|
| | representations | • Representing tridimensional objects based on bidimensional representations<br>• Building and deconstructing flat shapes and solids based on the conditions given<br>**Problem-solving:**<br>• Using geometric representations and establishing relations to solve problems |
| Development of surfaces of geometric bodies | Relates tridimensional objects and its properties with its respective development of surfaces | **Communication:**<br>• Interpreting information coming from practical measurement situations (furniture assembly, object construction)<br>• Describing procedures to build shapes and objects based on measures given<br>**Reasoning**<br>• Relating tridimensional objects and its properties with the respective development of surfaces<br>• Associating development of surfaces with its respective solids<br>**Problem-solving:**<br>• Using geometric representations and establishing relations to solve problems.<br>• Solving problems that require identifying patterns and regularities using geometric representations |

The design of the test (initially made up by 30 items) included learning evaluated by ICFES on communication, reasoning and problem-solving competences in spatial thinking of Saber tests for 3rd and 5th grades, conducted in the years 2012-2015. Likewise, its validation was subject to each item being within the interest domain, adequate for the object of the study and contextualized; its content validity was reviewed by three experts (mathematics graduates).

Once the items (22) were adjusted according to the experts' suggestions in terms of pertinence, adaptation, total number of questions and application time, the pilot test was conducted with 15 fifth-graders in one of the

Institution's branches; the results were analyzed with the SPSS 21 program, free version.

The Kuder-Richardson coefficient (1937) was used to measure the instrument's internal reliability; due to the item's characteristics, these are dichotomic, with correct or incorrect answers.

The value calculated by the coefficient was of 0.70 (high). Based on these results, the test kept the same items than those in the pilot test and the given test time was of 90 minutes.

On the other hand, to avoid the learning-derived bias that takes place when taking a test on several occasions, the order of the questions and answers was changed in the posttest.

In terms of the intervention in the EG (experimental group), firstly, researchers socialized the intervention with the students, explaining when and how it would take place, by whom, the object of the study and the competences to develop.

Thus, when implementing the work sessions where the strategy is applied in the EG, cooperative work groups were initially configured using three dynamics (assembling puzzles, color candy and 1 to 4 enumeration), taking into account the levels of intervention needed to work collaboratively that were described by Pujolas (2009). In the first level, uniting the group, students begin a knowledge and cooperation stage among them in order to achieve consensus; in the second, role rotation, students are taught how to work in teams, a group of experts makes rotations, and in this case, students group based on common roles, sharing experiences and then go back to the base group to socialize their learning. Afterwards in the development of the activities

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proposed in the didactic guidelines, the steps proposed by Polya's heuristic method (See example in Table 3) are applied. In the first step's group (understanding the problem), students work individually, socializing their understanding of the problem situation with their classmates; as a team they move on to the second step (devise a plan); followed by the third (carry out the plan) and fourth steps (look back). Then, the concept is collectively built and learning is followed-up through participation, development and socialization of the proposed activities.

**Table 3.**

***Proposal of a problem situation***

| Proposal of a Problem Situation | |
|---|---|
| How do the faces and prints of the geometric bodies look like when captured in a flat surface? What are the differences and similarities between them? | |
| 1. Understanding the problem<br>• What is a face? What do the prints of geometric bodies mean?<br>• What is the meaning of capturing? What do you understand by flat surface?<br>• Which geometric body are you working with? How do the faces of the cube look like? How does the cone look like? How do the faces or surface look like? What is the shape of the supporting part? What is the name of that part? | 2. Devising a plan<br>• Which strategies would you use to describe the solids' prints?<br>• How are solids' prints classified?<br>• What criteria needs to be considered?<br>• What characteristics did you find in the shape? |
| 3. Executing the plan<br>• Can you clearly see if the strategy you used is the correct one?<br>• How can you verify it?<br>Get on with the selected strategy | Looking back:<br>• What characterizes your rectangle?<br>• The geometric shapes captured in the page from the top to the bottom are the same?<br>• What characterizes the geometric shape captured from the top and bottom view in a prism? In a cube? In a pyramid? In the cylinder?<br>• What shape is the base of a: cube, cylinder, pyramid and cone? |

Also, the cooperative work methodology led to the development of social and thinking skills, as follows: respect for having the word, paraphrasing, inquiring on justification and reasons, asking for help to clarify, helping classmates and

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criticizing ideas (not people), with the purpose of favoring better learning environments in students.

## Analysis of the Results

The following is the analysis of the results obtained through the calculation of some measures with central tendency and significance levels, based on the research's objectives.

To classify the students' performance levels, ICFES' percentage distribution was used (see Table 4) according to the overall number of correct questions (22 questions) distributed by competences.

**Table 4.**

***Percentage distribution of performance levels***

| Performance level | Scale | Nº of questions | Percentage |
|---|---|---|---|
| Insufficient | 100-226 points | 0-9 | <45.4% |
| Minimum | 227-315 points | 10-13 | 45.4%, < 63.6% |
| Satisfactory | 316-399 points | 14-17 | 63.6% <80% |
| Advanced | 400-500 points | 18-22 | >=80% |

Source: Adaptation of the Interpretation Guide and Results Use of SABER tests for 3rd, 5th and 9th grades. Educational Establishments. Colombia 2015. Version 1.

### Analysis of the Results of the CG-EG Pretest

Once the results of the correct answers were analyzed, the percentage of control group (CG) and experimental group (EG) students located in the insufficient level was of 44% and 53%, respectively (Graph 1); meaning students have difficulty to solve problem situations that require identifying

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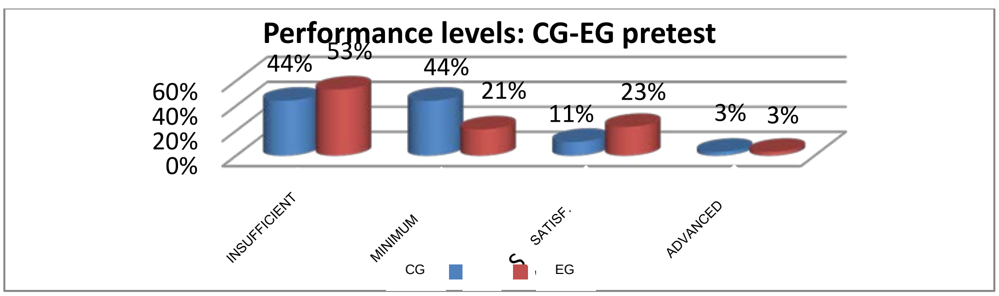
patterns and regularities in solids' use and representation and recognizing properties that are invariant when a plane transformation is applied.

44% and 21% of the CG and EG students are located in the minimum performance level, respectively; meaning they solve problem situations of lesser complexity related to the association of flat development of surfaces of solids and in deconstructing partial regions of flat shapes and solids. The remaining 14% of the CG is located in the satisfactory (11%) and advanced (3%) levels; 26% of the EG is located distributed in those two levels.

**Graph 1.**

***Pretest's performance level of fifth-graders at the Villa Cielo Educational Institution***




Source: compiled by the authors.

According to the results obtained (Graph 2) in the pretest, 36% and 41% of the CG and EG students, respectively, correctly answered questions related to the reasoning competence; meaning they represent, relate, build and deconstruct flat figures and solids based on given conditions.

Also, in the communication competence, approximately 61% of the students in each group fail to solve problem situations connected with identifying, interpreting and describing procedures for building and deconstructing shapes

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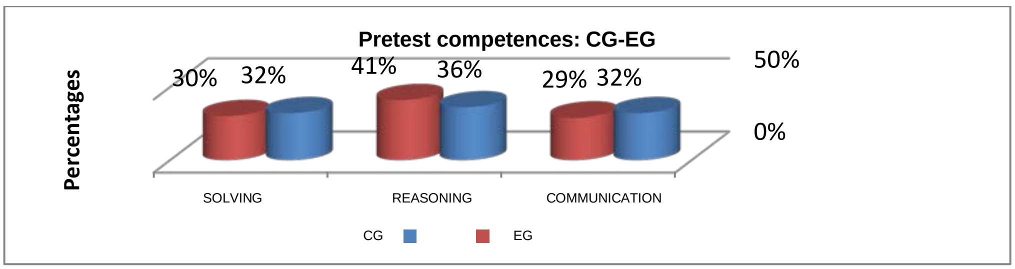
and objects. As illustrated in Graph 2, the average of correct answers was of 30%.

In terms of the problem-solving competence, it is clear that around 62% of the students fail to solve problems that require identifying patterns and regularities using geometric representations.

Graph 2.

*Percentages of competences in the pretest for each group*




Source: compiled by the authors.

**Analysis of the Results of the CG-EG Posttest**

In the analysis conducted on the correct answers provided by the students in the posttest (Graph 3), it is observed that 53% of the CG students are placed in the insufficient level; meaning they fail to solve problems of lesser complexity related to geometric solids; while there are no EG students in this level, which indicates positive changes in students' performance about geometric thinking in the geometric solids study unit.

In terms of the satisfactory and advanced performance level, the EG reaches 94%; meaning the students solve problems of higher complexity related to the comparison of attributes of shapes and solids based on its characteristics and to establishing relations between them.

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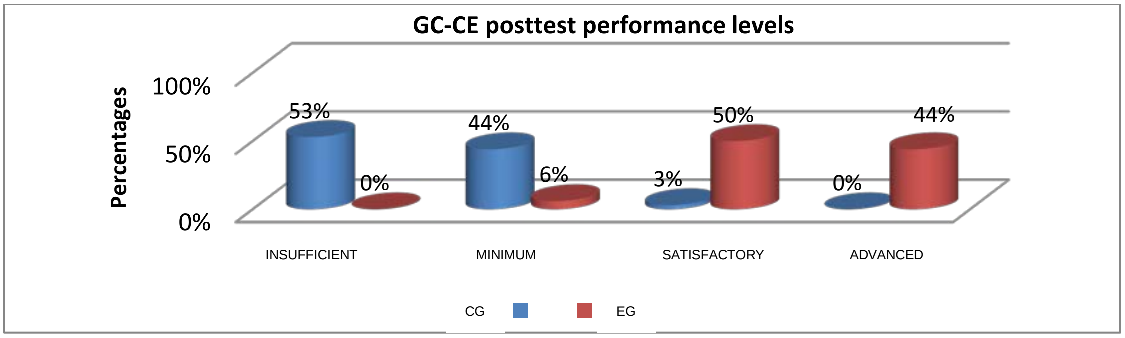
Graph 3.

*Posttest performance level of fifth-graders*




Source: compiled by the authors.

The aforementioned confirms positive changes in the dependent variable (geometric thinking), demonstrating that problem-solving applying Polya's model and the cooperative work methodology is an effective strategy that will serve other researchers as back-up to further content's didactic knowledge.

Table 5 illustrates the CG and EG's descriptive statistic, differences in the means after the strategy was applied were observed.

Table 5.

*CG-EG posttest descriptive statistic*

| GROUP | | Mean | Typical deviation | Typical mean error |
|---|---|---|---|---|
| Pos-Reas. | CG | 3.68 | 1.319 | .226 |
| | EG | 6.24 | .955 | .164 |
| Pos-Comm. | CG | 2.82 | 1.218 | .209 |
| | EG | 5.50 | 1.619 | .278 |
| Pos-Reas. | CG | 2.38 | 1.155 | .198 |
| | EG | 5.09 | 1.055 | .181 |
| Pos-Total. | CG | 8.94 | 2.361 | .405 |
| | EG | 16.82 | 2.302 | .395 |

Source: SPSS 21. Free version.

Analyzing both groups, Table 3 shows changes in the means of the CG and the EG, which provided support when finding differences among groups,

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meaning, EG students solved problem situations related to associating development of surfaces and its respective solids, and to deconstructing flat figures and solids into partial regions, which indicates that the strategy of Polya's heuristic method was effective.

On the other hand, the Levene test (Table 6) was used to determine the population's characteristics. It shows that groups are homogeneous among them (p>0.01) and that performance in each competence and overall between the CG and the EG is different (p<0.05), thus indicating a difference in performance after the strategy application that favors the EG, where averages significantly increased, as evidenced in Table 2.

Table 6.

*T test on comparison of CG-EG competences*

| | | **Levene test** | | **T test for measure equality** | | |
|---|---|---|---|---|---|---|
| | | **F** | **Sig.** | **T** | **Sig. (bil.)** | **Typical difference error** |
| Pos-Reas. | Equal variances have been assumed | 5.063 | .028 | -9.160 | .000 | .279 |
| Pos-Comm. | Equal variances have been assumed | 6.098 | .016 | -7.703 | .000 | .347 |
| Pos-Reas. | Equal variances have been assumed | .388 | .535 | -10.085 | .000 | .268 |
| Pos-Total | Equal variances have been assumed | .129 | .721 | -13.938 | .000 | .566 |

Source: SPSS 21. Free version.

## Analysis of the Results of the CG Pretest-Posttest

Table 7 shows this group's descriptive statistic, observing changes in means before and after certain time (three months).

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Table 7.

*CG pretest - posttest related sample*

| GROUP | | | Mean | Typical deviation | Typical mean error |
|---|---|---|---|---|---|
| CG | Par 1 | Pre-Reas. | 3.79 | 1.666 | .286 |
| | | Pos- Reas. | 3.68 | 1.319 | .226 |
| | Par 2 | Pre-Comm. | 3.32 | 1.319 | .226 |
| | | Pos-Comm. | 2.82 | 1.218 | .209 |
| | Par 3 | Pre- Reas. | 3.35 | 1.454 | .249 |
| | | Pos- Reas. | 2.38 | 1.155 | .198 |
| | Par 4 | Pre-Total | 10.47 | 3.578 | .614 |
| | | Pos-Total | 8.94 | 2.361 | .405 |

Source: SPSS 21. Free version.

On the other hand, Table 8 indicates there were no differences in this group's reasoning and communication competences (p>0.05), in contrast with the problem-solving competence which does show before and after differences, a situation that might have originated in the teacher's strategies, lack of innovation in resources and materials to promote students' autonomy and interest in learning, lack of exploration in the students' prior knowledge, coinciding with Lastra's (2005) description.

Table 8.

*CG pretest - posttest related sample*

| Group | | Mean | Typical deviation | Typical mean error | T | gl | Sig. (bil) |
|---|---|---|---|---|---|---|---|
| CG | Pre-Reas. –Pos- Reas. | .118 | 2.086 | .358 | .329 | 33 | .744 |
| | Pre-Comm. -Pos-Comm. | .500 | 1.728 | .296 | 1.688 | 33 | .101 |
| | Pre- Reas. - Pos- Reas. | .971 | 1.915 | .328 | 2.956 | 33 | .006 |
| | Pre-Total - Pos-Total | 1.529 | 4.266 | .732 | 2.091 | 33 | .044 |

Source: SPSS 21. Free version.

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**EG Pretest-Posttest**

Table 9 shows the EG's descriptive statistic and the means' value to identify competences' behaviour before and after the intervention.

Table 9.

*EG posttest descriptive statistic*

| GROUP | | | Mean | Typical dev. | Typical mean error |
|---|---|---|---|---|---|
| EG | Par 1 | Pre-Reas. | 4.35 | 1.228 | .211 |
| | | Pos- Reas. | 6.24 | .955 | .164 |
| | Par 2 | Pre-Comm. | 2.97 | 1.381 | .237 |
| | | Pos-Comm. | 5.50 | 1.619 | .278 |
| | Par 3 | Pre- Reas. | 3.12 | 1.493 | .256 |
| | | Pos- Reas. | 5.09 | 1.055 | .181 |
| | Par 4 | Pre-Total | 10.44 | 2.776 | .476 |
| | | Post-Total | 16.82 | 2.302 | .395 |

Source: SPSS 21. Free version.

On the other hand, Table 10 indicates changes ($p<0.05$) in students' performance for each competence, meaning students solve problem situations of higher complexity relating geometric solids.

Table 10.

*EG pretest - posttest related sample*

| Group | | Mean | Deviation | Typical mean error | 95% I with difference | | T | Gl | Sig. (bil) |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | Inf. | Sup. | | | |
| EG | Pre-Reas. -Pos-Reas. | -1.882 | 1.552 | .266 | -2.424 | -1.341 | -7.070 | 33 | .000 |
| | Pre-Com.-Pos -Com | -2.529 | 2.149 | .369 | -3.279 | -1.779 | -6.862 | 33 | .000 |
| | Pre-Reas. - Pos- Reas | -1.971 | 1.586 | .272 | -2.524 | -1.417 | -7.247 | 33 | .000 |
| | Pre-Total - Pos-Total | -6.382 | 2.850 | .489 | -7.377 | -5.388 | -13.058 | 33 | .000 |

Source: SPSS 21. Free version.

## Discussion

This framework assumes a model centered in problem-solving with the intention to promote didactic strategies based on proposing problem situations to develop competences that foster meaningful learning in the students. Thus, the didactic strategy of Polya's heuristic method is used to develop the competences of reasoning, communication and problem-solving in geometric thinking.

Observing the posttest results, a match with those obtained in research by authors such as Boscan and Klever (2012), Aguilar and Navarro (2012) appears, these authors concluded that according to Polya, problem-solving favors students' learning.

Consequently, in terms of geometric thinking competence development in the solids study unit, this research ratifies what was stated by Guillen (2010) and Osorno (2014) highlighting the importance of recognizing solids to develop geometric thinking, and also that was said by Blanco (2013) in terms of the influence of didactics and social aspects to understand spatial geometry.

Applying the cooperative work methodology in one of the EG, the results following the intervention are similar to those found in the research by authors Ferreiro (2007) and Gonzalez and Garcia (2007) who affirm that cooperative learning increases and diversifies communicative skills (understanding, explaining, asking and answering) with the correct language and favoring students' learning.

The valuation of the tests conducted before and after the intervention to assess the efficacy of the didactic strategy given the general objective was determined through statistic tests. These found that the pretest showed no significant differences in the students' performance in each competence, while the posttest did show significant differences among the groups in favor of the EG, leading to the conclusion that Polya's heuristic method develops skills and dexterities in students to classify, interpret and solve problem situations related to geometry and other fields. This confirms Polya's (1981) words affirming that solving a problem means creating posterior skills to solve any kind of problem.

In the EG, students evidenced better performance solving problem situations in comparing, identifying and representing tridimensional objects, due to a responsible and constructive interaction between all of the group members to clearly and fluently communicate (orally or in writing) the results of the problem, coinciding with was stated by Barnett *et al.* (2003).

The aforementioned implies that the didactic intervention had positive results in competences' development in the EG, where important and satisfactory progress was observed in connection with performance levels, this indicates that the students overcame prior difficulties and unlocked a more advanced level, solving situations of more complex nature.

Based on that fact, it can be assumed that the proposed activities and the development of work sessions were an opportunity to develop mathematical competences, strengthening related students' processes.

## Conclusions

The results obtained by the research have allowed verifying the objectives set out at the beginning of the study, and have allowed reaching conclusions regarding the incidence of the designed and applied didactic intervention, taking into consideration Polya's heuristic method for geometric thinking; in summary, it was found that:

In the application of the intervention with Polya's heuristic method, a pretest was conducted to identify the students' weaknesses; a subsequent analysis of these weaknesses led to the design and application of learning guides based on Polya's heuristic method on geometric solids. The development of each learning activity evinced that students favor construction of concepts when interacting on their own and in their social environment, thus obtaining improvement in leaning, as it was exposed by Barnett *et al.*, (2003).

After the intervention with Polya's heuristic method, the performance level of the competences of reasoning, communication and problem-solving satisfactorily increased in the EG, indicating the proposal's efficacy.

Additionally for the EG, after the intervention it was clear that the cooperative work methodology strengthens the communication competence even more, due to the fact that this strategy intends to develop skills that tend to collectively build learning.

Likewise, the didactic strategy of Polya's heuristic method plus cooperative work aided in understanding the groups' dynamic in the classroom, which led to interventions that improved the didactic strategy's implementation, aside to reflections on the processes taking place in the classroom regarding each one's role. This strategy can be implemented as helpful tool for the teacher's task in any school grade and in different knowledge areas, and it can also become a reference for further research.

The implementation of the didactic strategy in an academic term was advantageous to improve the EG's skills and competences on geometric thinking; feeling as active parties of the process and being able to express their thoughts helped the groups' students' learning become meaningful while fostering knowledge building. In that way, the teacher guides and accompanies the didactic work.

Problem-solving may be successfully used as a strategy to develop competences applying Polya's (1981) heuristic method's problem-solving stages and adapting them to respond to current contexts and to the population's educational needs.

In terms of the curriculum, the possibility of including the didactic strategy of Polya's heuristic method and cooperative work methodology in area plans to develop didactic units should be considered; applied responsibly and without improvisation, these support pedagogical practices and students' learning.

Finally, it was also confirmed that this research's didactic strategy was necessary to really get to know and solve problems related to mathematical objects, and with geometry in particular, to achieve a complete curricular proposal based on Polya's heuristic method for the students of the Villa Cielo Educational Institution in the municipality of Monteria.

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# Annex 1

*(Refer to the original test below for each question's images)*

Full name: ___________________________________________

Date: ______________________________________________

Grade: _____________________________________________

Group: _____________________________________________

This test contains 22 single-answer multiple-choice questions, you have 90 minutes to complete it. Read each question and answer carefully, answer one between A, B, C or D.

1. Andres is looking at a solid's front view:

Which shape is Andres seeing?

A. B. C. D.

2. The following tower was built using blocks such as this one:

How many blocks were used in total to build the tower?

A. 8 B. 9 C. 16 D. 17

3. Camilo looked at a solid from different positions. This was what he saw:

Side Front Top

Which solid is Camilo seeing?

A. B. C. D.

4. Beto used blocks such as this one to build the following solid:

How many blocks did Beto use to build the solid?

A. 4 B. 2 C. 6 D. 3

5. The following solid must be assembled using two pieces:

Which pair of pieces can be used to assemble the solid?

A. B. C. D.

6. Pedro, Ana and Marcela are looking at a solid built with three equal cubes, Pedro is looking at it from the top, Ana from the side and Marcela from the front.

Which figure shows Pedro's view of the solid?

A. B. C. D.

7. Look at the tower in Figure 1.

The tower was built using these three blocks.

How many small blocks were used to build the tower in Figure 1?

A. 4 B. 7 C. 8 D. 13

8. Francisco used a carboard mold to build a box as the one shown:

Which mold did he use to build the box?

A. B. C. D.

9. Leonardo wants to build a solid as this one using two blocks.

Which pair of blocks can he use to build the solid?

A. B. C. D.

10. Look at the tower and some of its measurements.

Which group of blocks can be used to build a tower with the same measurements?

A. B. C. D.

11. Oswaldo used two different blocks to assemble a solid as this one:

Which group of blocks did he use to assemble the solid?

A. B. C. D.

12. Marcela used two different blocks to build a solid as this one:

Which group of blocks did she use to build the solid?

A. B. C. D.

13. Which of the following is the development of the surface of a pyramid?

A. B. C. D.

14. Which of the following blocks did Mauricio and Carolina use to build the cube?

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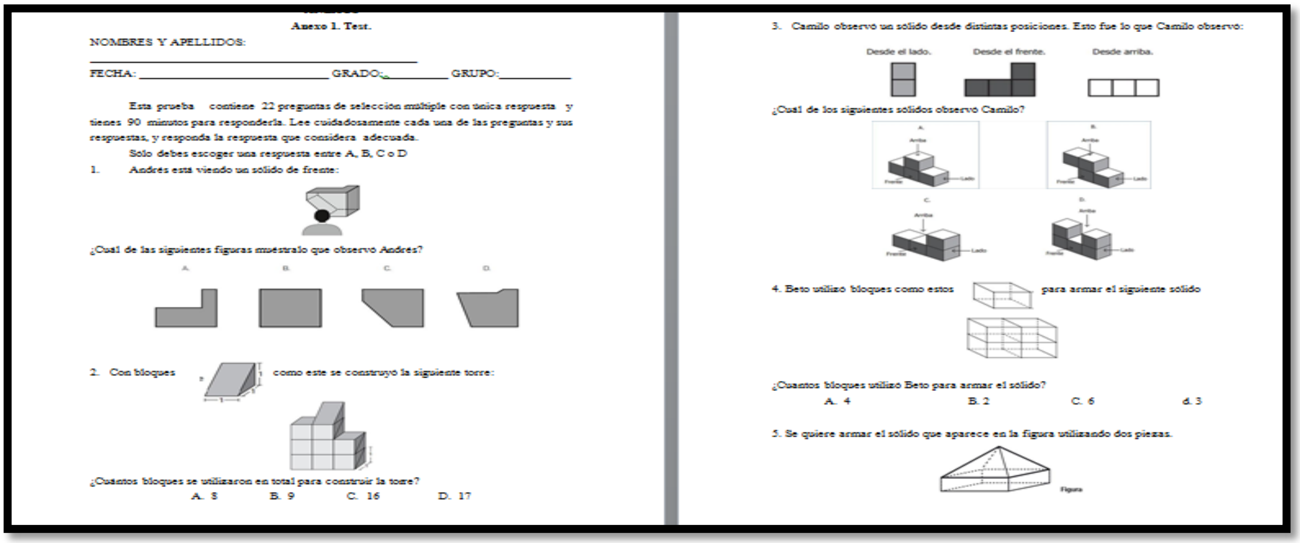

**Anexo 1. Test.**

NOMBRES Y APELLIDOS:
______________________________________________

FECHA: ______________________ GRADO:__________ GRUPO:___________

Esta prueba contiene 22 preguntas de selección múltiple con única respuesta y tienes 90 minutos para responderla. Lee cuidadosamente cada una de las preguntas y sus respuestas, y responda la respuesta que considera adecuada.

Sólo debes escoger una respuesta entre A, B, C o D

1. Andrés está viendo un sólido de frente:

¿Cual de las siguientes figuras muéstralo que observó Andrés?

A. B. C. D.

2. Con bloques como este se construyó la siguiente torre:

¿Cuántos bloques se utilizaron en total para construir la torre?

A. 8 B. 9 C. 16 D. 17

3. Camilo observó un sólido desde distintas posiciones. Esto fue lo que Camilo observó:

Desde el lado. Desde el frente. Desde arriba.

¿Cuál de los siguientes sólidos observó Camilo?

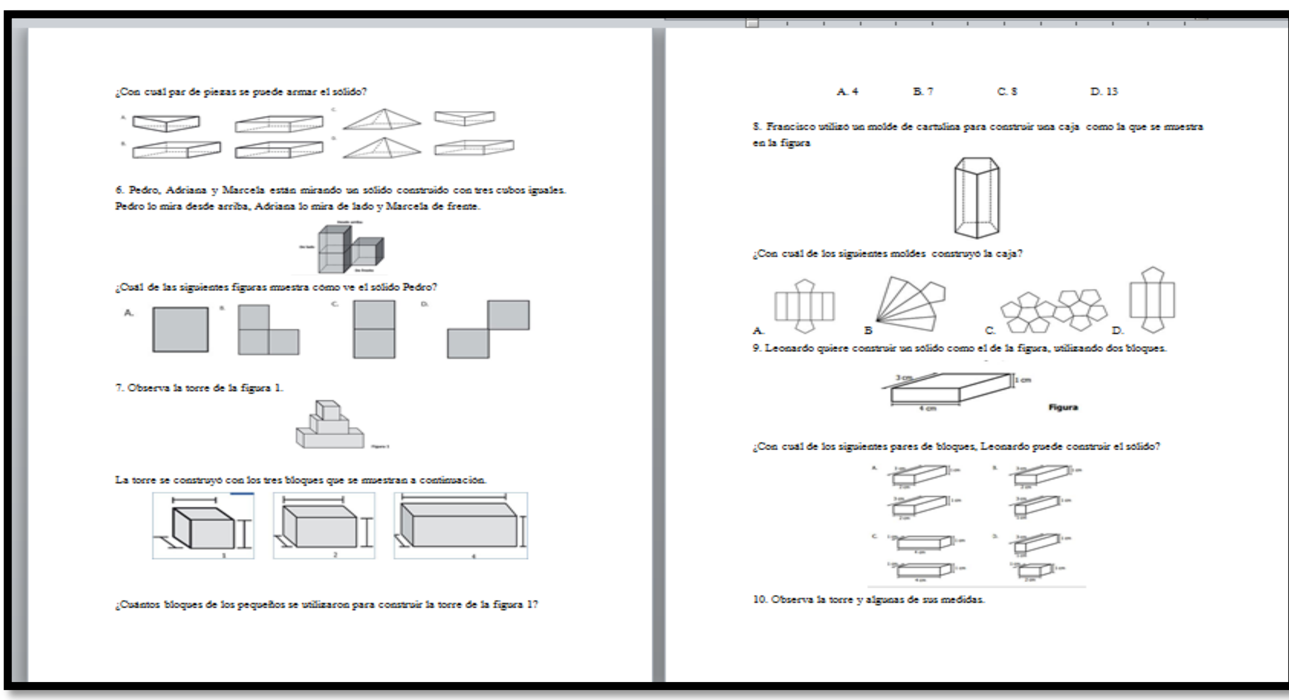
4. Beto utilizó bloques como estos para armar el siguiente sólido

¿Cuantos bloques utilizó Beto para armar el sólido?

A. 4 B. 2 C. 6 d. 3

5. Se quiere armar el sólido que aparece en la figura utilizando dos piezas.

Figura


¿Con cuál par de piezas se puede armar el sólido?

6. Pedro, Adriana y Marcela están mirando un sólido construido con tres cubos iguales. Pedro lo mira desde arriba, Adriana lo mira de lado y Marcela de frente.

¿Cuál de las siguientes figuras muestra cómo ve el sólido Pedro?

A.

7. Observa la torre de la figura 1.

La torre se construyó con los tres bloques que se muestran a continuación.

¿Cuántos bloques de los pequeños se utilizaron para construir la torre de la figura 1?

A. 4 B. 7 C. 8 D. 13

8. Francisco utilizó un molde de cartulina para construir una caja como la que se muestra en la figura

¿Con cuál de los siguientes moldes construyó la caja?

A. B C. D.

9. Leonardo quiere construir un sólido como el de la figura, utilizando dos bloques.

3 cm 1 cm 4 cm Figura

¿Con cuál de los siguientes pares de bloques, Leonardo puede construir el sólido?

10. Observa la torre y algunas de sus medidas.

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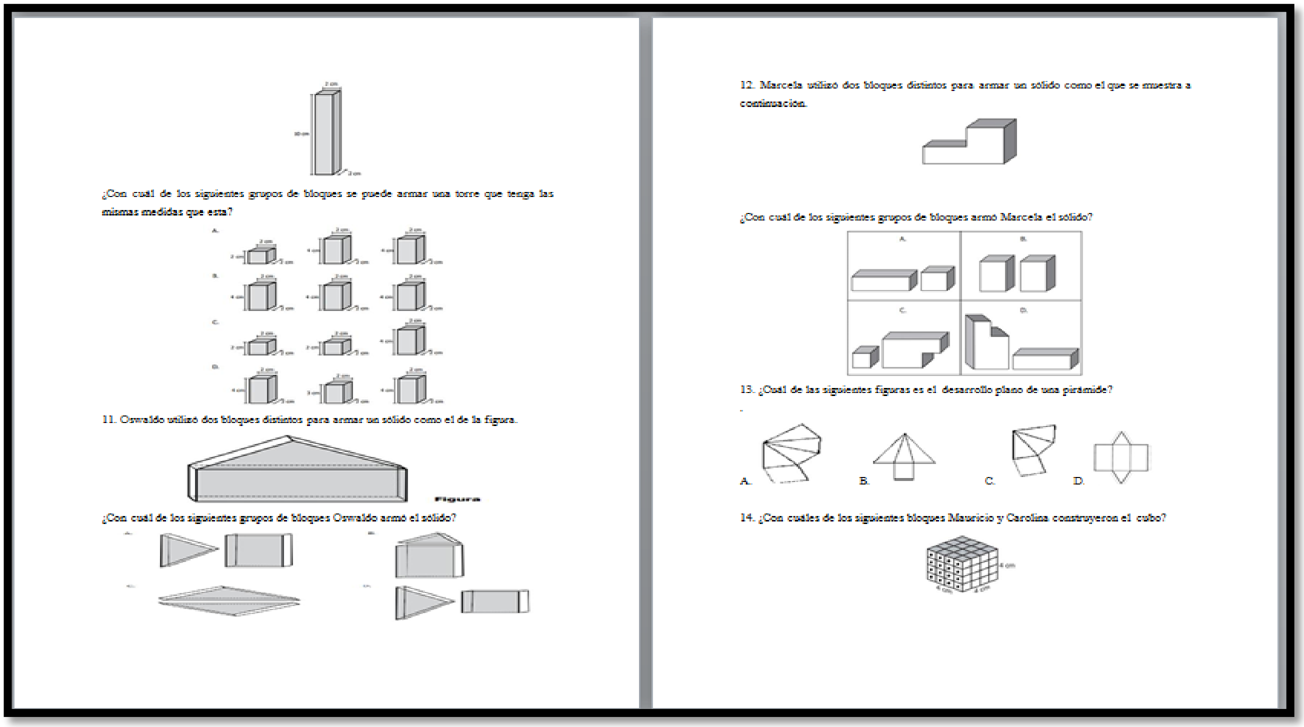
¿Con cuál de los siguientes grupos de bloques se puede armar una torre que tenga las mismas medidas que esta?

A.

B.

C.

D.

11. Oswaldo utilizó dos bloques distintos para armar un sólido como el de la figura.

Figura

¿Con cuál de los siguientes grupos de bloques Oswaldo armó el sólido?

12. Marcela utilizó dos bloques distintos para armar un sólido como el que se muestra a continuación.

¿Con cuál de los siguientes grupos de bloques armó Marcela el sólido?

A. B. C. D.

13. ¿Cuál de las siguientes figuras es el desarrollo plano de una pirámide?

A. B. C. D.

14. ¿Con cuáles de los siguientes bloques Mauricio y Carolina construyeron el cubo?