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SEMIOTIC REPRESENTATION OF THE NOTION OF FUNCTION: IDEAS OF HIGH SCHOOL GRADUATES ENROLLED IN UNIVERSITY

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Abstract

This article is a report of a larger research aimed at assessing the changes or modifications in the students' skills to articulate diverse registers of semiotic representation regarding the notion of function in students enrolled in the subject of differential calculus of two engineering programs in a public university. The research's reference framework is based on the work of several researchers, especially by Duval and Hitt. The methodology is quantitative and of descriptive nature. Data was collected with an eight-item test including diverse registers of semiotic representation regarding the notion of function. This test was applied at the beginning of the experiment, followed by a pedagogical intervention on the concept of function based on diverse registers of semiotic representation, after which a new test was applied. The results show ideas observed in the students at different moments of the semester, comparing the data obtained in the pre-test and the post-test. The students' notion of function does not correspond with a formal definition; instead, the notion manifests a series of conceptual variations that, in some cases, are closer to an intuitive notion.

Key Words: function, semiotic representations, articulation of registers, conception.

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

Introduction

Currently, universities have to deal with unsatisfactory learning of mathematical concepts by students (leading to a considerable number of them failing mathematics subjects due to underperformance), this has become a significant problem, and it has been the object of attention of teachers' academic work and of mathematical education researchers alike. In the particular case of the Engineering Department of Universidad Francisco de Paula Santander (hereinafter, UFPS), specifically, the subject of differential calculus, a considerable number of students show problems in learning mostly in the unit that includes the concept of *function*, despite the fact that this concept is not new to the students, since it has been previously taught in elementary and high school.

According to the Basic Standards of Mathematics Skills (Ministry of National Education, 2006), one of the overall processes of mathematical activity is the modelling processes and phenomena of reality. Therefore, mathematics learning may be demonstrated by the student's capacity to generate schemata based on everyday scientific and mathematical situations of iterative nature, and by the possibility of mentally reconstructing said schematized situations. The modelling process is necessary to become mathematically competent, this process is specifically developed in the logical and mathematical thinking; the latter encompasses five types of thinking that are included in the Curricular Guidelines (Ministry of National Education, 1998), of which two, variation and algebraic and analytical systems, are relevant to this research. These two types of thinking relate to knowledge, perception, identification and characterization of variation and changes in different contexts, as well as to description, modelling and representation of different symbolic systems or registers of verbal, iconic, graphic or algebraic nature.

As previously stated, the concept of function is mandatory in almost every school curriculum, from elementary (aimed at building different roads and significant approaches to understand and use the concepts and procedures of function, and its analytical systems regarding numerical and algebraic calculus) to high school and higher education (in differential and integral calculus subjects).

Despite the relevance of this concept, and of others of calculus, different research has proven that students in almost every level have problems understanding it, both in high school and in higher education.

Internationally, Ferrari (2001) and Prada, Hernandez and Ramirez (2016), describe some research conducted on different positions and difficulties regarding the concept of function; the image and definition of the concept (Vinner, 1992, 2002; Tall, 1992, 2002; Eisenberg, 2002), APOE theory (Dubinsky, 2002), the tool-object dialectics and context play (Douady, 1986, 1995, 1996), articulation between registers and semiotic representations (Artigue, 1995; Duval, 1988, 1993, 2004, 2006), epistemological obstacles and acts of understanding (Sierpinska, 1992), and variation thinking and language (Cantoral and Farfan, 1998; Farfan, 1997), among many others. Due to the importance of its point of view to this research, Hitt's research on function focused on visualization (1994; 1998; 2003a, 2003b, 2003c) is set apart from the others.

In the Colombian context, Prada, Hernandez and Ramirez (2016) highlight the research associated to the concept of function: Garzon (2015), Sanchez, Martinez and Coronado (2015), Garcia (2013), Ospina (2012) and Rojas (2012, 2014).

Mastering algorithmic processes is not enough to acquire concepts of calculus. However, mathematics teachers often limit their instruction to an algebraic-like type of teaching (Hitt, 2003a, 2003b), which certainly results in limitations in its comprehension. According to Hitt, the tasks of connecting different representations of a concept (achieved by exercises of conversion between representations) are not considered fundamental to the construction of mathematical learning by many teachers, moreover, conversion exercises are often minimized by teachers in terms of its relationship with the concept of function. However, Hitt (2003b) thinks that conversion exercises would encourage better understanding of functions and allow the development of visualization processes.

The mathematical object of function may be represented analytically (algebraically), in a tabular or graphic way, or in natural language. Conversion allows an articulation between the registers of representation and teaching, it is the result of conceptual understanding and any difficulty that arises, and it also indicates that the constriction of the concept has not ended yet. According to Duval (2006, p. 166), “it is the first threshold in comprehension of mathematics learning”.

Duval (2004) states that at least two characteristics of cognitive action are involved in the development of mathematical skill. On the one hand, using diverse registers of semiotic representation, some of which have not been developed specifically to apply mathematical models. On the other hand, mathematical objects are not always accessible through visualization. Based on that statement, the author asks the following: how can we learn to change the register? And how can we learn not to confuse the object with its representation? Since mathematical activity usually relates models and conversions, the differentiation of registers of representation, as well as coordination and conversion between them, constitute critical points for learning.

In that regard, Hitt (2003c), claims that Duval considers conversion exercises between semiotic representations of a mathematical object as one of the core aspects in his research on the construction of concepts, and distinguished the existence of noticeable differences in the construction of concepts on whether it is a perspective that is linked to everyday life or a perspective that chooses the mathematical concept as a point of view. Indeed, mathematical objects (lineal equations with two variables, for this particular research) are accessible only through semiotic representations, which differ from everyday life, where representations are physical objects.

This is why understanding the content of a concept relies on the coordination of at least two registers of representation, becoming evident in the speed and naturalness of the cognitive action of conversion. For instance, the concept of lineal function is an

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

abstract entity, it has several semiotic representations to ease its understanding, however, according to Godino (2003, p. 53),

> the object represented may vary depending on the semiotic representations in the learning of the lineal function concept as per the context or use of the representation: in the case of a Cartesian plane, it may represent a function or the solution set of an algebraic equation.

D'Amore (2009), proposes that one of the difficulties in the representation of mathematical objects is the transit of a concept through its diverse representations. The conceptual acquisition of an object necessarily goes through the acquisition of one or more semiotic representations.

> Therefore, the construction of mathematical concepts closely depends on the capacity to use more registers of semiotic representations of said concepts:
>
> 1. Of representing them in a given register.
>
> 2. Of dealing with said representations within the same register.
>
> 3. Of converting said representations from a given register to another. (D'Amore, 2009, p. 158).

For Prada, Hernandez and Ramirez (2016), in teaching calculus' fundamental concepts, the courses are developed around the study of properties and characteristics associated to the concept of function, such as types of functions, domain, range, derivative of a function, operations between functions; the graphic representation of functions is reduced to drawing the graph given its algebraic expression, following previously determined steps: a chart of values (almost always with positive and integer values), a representation in the Cartesian plane and joining them through a line (assuming the continuity of the values without having evaluated them) that might be a straight line or a curve, without didactic sequences aimed at addressing the construction of the concepts and the articulation of the diverse registers of representation. Moreover, teachers generally offer students the mechanical tools to conduct some procedures and techniques that allow them to solve standardized exercises and problems, leaving out the context of reality and the importance of manner and moment of presenting the topic at hand in terms of concrete situations. However, the fact that a student can undertake the mechanics more or less correctly does not imply that he/she has gained satisfactory understanding of its underlying notions. On the other hand, mastering the procedures without understanding the notions that support it may be because traditional university teaching tends to favor the algorithmic practice of registers of representation putting evaluative processes and mechanical replication of certain solution algorithms ahead of understanding and applying the concepts.

This framework proves the importance of the use of representations when teaching the concept of function, in order for students to develop the skill of manipulating mathematical objects in their diverse registers. Consequently, the objective of this research is to assess the changes or modifications in skills of students enrolled as freshmen in the subject of differential calculus in the Engineering Department at UFPS, to articulate diverse registers of semiotic representation of the notion of function.

Method

The methodology used in this research is quantitative. It is of descriptive nature. Based on the answers the groups provided to the tests, as well as on individual arguments and comments by some randomly selected students, we analyzed the ideas deriving from the application of an articulation of diverse registers of semiotic representation in teaching the concept of function among freshmen enrolled in differential calculus in the Engineering Department at UFPS in the city of Cucuta (Colombia) during the first term of 2015 in the programs of Systems Engineering (with high-quality certification) and Electromechanical Engineering (with qualified registration). The quality conditions are defined by the National Council of Certification (CAN, for its Spanish acronym), affiliated to the National System of Certification of the Ministry of National Education (MEN, for its Spanish acronym.

A non-probabilistic sampling process was applied with the convenience sampling technique, since the intention was to identify the possible existence of academic differences among students enrolled in two different academic programs of the Engineering Department (which have different quality conditions). A total of 42 and 41 students of the Systems Engineering and Electromechanical Engineering participated, respectively. Both groups were made up by students of both genders (although there was a predominance of males), averaging 17 years of age. Though it has not been considered an explanatory variable, it is important to mention that 95% of the sample's subjects graduated high school in 2014 and come from families with socio-economical levels 2 and 3.

An instrument consisting of nine items with diverse registers of representation of the concept of function (Cf. Annex 1) was designed. It was contextualized and based on Hitt's (2000) work regarding functions in context. This allowed an exploration of different registrations in terms of functions, considering its different theoretical elements.

The instrument was applied to the same sample in two different moments with the objective of demonstrating the effects of a pedagogical intervention process. Each test took 120 minutes. After the diagnostic (pre-test) was applied, a series of difficulties regarding diverse elements associated with the topic of functions were defined. Afterwards, the teachers intervened the groups for four weeks, with an intensity of four hours a week, drilling workshops aimed at reinforcing the articulation of different registers of representation and designed to tackle the difficulties identified in the pre-test. After the intervention, each teacher applied the instrument (post-test) and the data was processed in a descriptive way to compare the results of both tests, taking into account the academic program in which the students were enrolled. A sample of six

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

students was selected for the case study, which had the objective of getting to know the arguments used for the proposed questions.

Results

The answers provided by the students were classified as correct or incorrect. Table 1 shows the criteria used to grade each answer as support and rationale of the processes conducted by students in answering to the given problems. The Table also includes the organized results and the ratio of correct answers in both moments in which the test was applied.

Table 1. Results of the Instrument Applied in Both Moments.

Item	Type of articulation	Solution key	Correct answers pre-test	Correct answers post-test	Pre-test findings
1	From graphic to graphic	Apply the vertical line criteria to verify if it is a function.	12%	94%	Strong belief in the idea that every parabola is a function, to the extent of suggesting that under the ***horizontal line criteria,*** the graph represents a function (in the case of parabolas whose main axis is parallel to the X axis.

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

2	From everyday language to graphic or algebraic language	Validate the existence of piecewise-defined or hybrid functions based on a descriptive formulation.	4%	72%	Belief in the idea that the existence of the function supposes its trajectory must be continuous, and it must have a single algebraic expression, being unaware of the piecewise-defined or hybrid functions.
3	From graphic to algebraic language	Asses the values of points in a graph in different algebraic expressions and verify the principle of equality.	18%	92%	Identification of ordered pairs and assessment of those values in an algebraic expression, demonstrating the use of integer and positive values only; afterwards, they join them and assume the continuity in the real number set.
4	From algebraic to graphic language	Asses if the domain and range intervals expressed in formal language are	4%	89%	Difficulty to identify and understand and choose formal mathematical language, consequently, random answers demonstrate

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

		reflected in a given graph.			the confusion between domain and range.
5	From tabular to graphic	Locate ordered pairs in the Cartesian plane, taking into account the scale independence between the axis.	22%	78%	Acknowledgement of a lineal relation between the *x* and *y* values with presence of inadequate scales in the axis, thus he resulting graph is an incorrect representation.
6	From algebraic to graphic language	Assess diverse values for the *x*, identify for which value it is note defined and associate it to a vertical asymptote.	5%	94%	Construction of the graph aided on the tabular register and posterior translation to the Cartesian plane. Evidence that a maximum of three values (integer and positive) are used to construct the table. After the points are located, they are joined assuming a continuity of the graph without

					assessing it in many of these values.
7	From graphic to algebraic language	Apply the basic principles of graph representation (vertical and horizontal displacements and symmetry), based on a known function.	3%	95%	Manifestation of the students of the fact that they were not taught that it is possible to determine a graph's associated algebraic expression based on the graph. Therefore, the procedure is too complicated; those who do it, propose an expression evaluating some points of the graph and verifying the equality.
8	From everyday to algebraic language	Solve the formulation of a situation in context that requires combining several algebraic and geometrical concepts.	2%	96%	Overall, the activity is hard for almost all the students, who argue that they do not know how to address this type of situations. Some propose solutions that were nonsensical.

Source: compiled by the authors based on the research process.

The results obtained in both academic programs (Systems and Electromechanical Engineering) fail to offer enough significant differences to justify discriminating the averages, which leads to the first finding of the research: the fact that there are no representative differences between the students enrolled in a high-quality certification (which demands outstanding results in the *Pruebas Saber 11* as an admission criteria) and the students with inferior results in the same test. This demonstrates that getting good scores in standardized tests does not guarantee proper understanding of mathematical concepts. This finding is aligned with the statement of Santos and Vargas (2003, p. 10) “having good scores in this evaluations does not necessarily reflect deep understanding of the subject”, likewise with that of Popham (2001, p. 4) “students’ scores in these tests are not an accurate indication of the efficiency of teaching” and “any inference about educational quality that is based on students’ accomplishments in standardized tests tends to be invalid”.

Other findings deriving from the results of Table 1 are as follows:

The concept of function is essential for the development of the concepts taught in the calculus subject; therefore, full understanding of it is required to master complex mathematical notions, as stated by specialists such as Duval and Hitt.

Ideas regarding diverse difficulties identified in this study do not differ much from those found by other research. Indeed, findings confirm those of other works, which show that despite being conducted in different educational settings, the research’s underlying problem is the same: the complexity of the topic in itself and the inappropriate pedagogical process (which instead of facilitating total understanding, potentially increases comprehension difficulties).

In the test, students expressed they acquired and developed the concept of function in school, and that it did not generate difficulties back then. To that effect, they mentioned that their teachers provided them with an algebraic expression, built a value chart of maximum four values for the independent variable, located those ordered pairs in the Cartesian plane and then joined the dots with a line that was either straight or curve, depending on the distribution of the points. The aforementioned coincides with the findings of Dreyfus (2002), who explains that students limit themselves to working only with the algebraic representation of the function, failing to comprehend the idea or the need to transfer the knowledge in the shape of a chart and/or graph to the equation. Moreover, this finding strengthens the existing fracture between algebra and calculus, which had previously been addressed by Artigue (1995).

Students' opinions lead to establishing responsibilities throughout the process; clearly, the teacher largely influences students' development. Many of them have actually reduced the concept of function to a very minimalistic view, unaware of its importance on other mathematical concepts, using algebraic registers in excess and ignoring the articulation processes of diverse registers; this it to say that many mathematics teachers, in their teaching practice, fail to incorporate activities that conduct students to establishing connections between diverse registers of representation and to provide the same degree of importance to each register within the classroom, agreeing with what was stated by Zuñiga (2009). In fact, given its abstract origins, it is a fundamental component in the construction of mathematical knowledge. In the specific case of the concept of function, the teacher must give enough importance to the development of mathematical visualization, which in turn is essential to understanding and acquiring complex mathematical concepts in a significant way.

The results, as mentioned, show that the students' difficulties have a direct relation with the teaching system, particularly in terms of how the teacher presents and structures content. On this matter, Cuestas (2007) proposes that a didactic unit based on the students' difficulties leads to a type of teaching that boosts learning, hence, the development of this type of research provides background to our local context, which in turn contributes with guidelines for teaching planning processes. Going back to the analysis of the results obtained, despite the large difference in performance of students in the two moments of the test, the results do not come as a surprise. Throughout the pedagogical intervention, teachers emphasized on the coherent use and articulation of diverse registers of semiotic representation, which unavoidably resulted in a higher level of understanding of the concept of function. This proves that teaching methodologies that step away from what is traditional draw better results. In this regard, Planchart (2002) says that modeling activities and thoughtful use of technological tools that efficiently encourage different representations promote the construction and perfection of the concept of function in the student.

Students' conceptual variations in terms of the concept of function are an indicator of a process that is under construction, intended to come close (although not yet) to the curricular concept. Among them, the following standout: the identification of function with that of ordered pairs and the association of a single correspondence between pairs of elements, and even between pairs of sets. Difficulties understanding the concept of function are mostly linked with the correspondence of the single value, which assumes a reductive view that thinks there is a constant correspondence by which each ***x*** value corresponds to a single value in ***y***, and that it cannot be repeated. Therefore, and due to a lack of algebraic skills, constant functions are unknown, they are acknowledged in the Cartesian plane but not in a Sagittal plane. To discern if a given expression is or not a function, it is necessary to rely in visual or graphic support, rather than on analytical support. Also, a graph represents a function if it is continuous, understood as a synonym of sequence or non-interruption. This disregards the existence of piecewise-defined or hybrid functions. In sum, the notion of function is basically understood by students in terms of relation, only some analyze the characteristics of a function, and a few understand its formal definition. This results constitute the evidence of the difficulty of some students to establish the relation between the variables of the function, which agrees with findings of prior research, such as that of Even and Bruckheimer (1998). Other students manage to define the concept of function, but fail at deciding if a graph represents it, a finding that was also determined by the work of Leinhardt, Zaslavsky and Stein (1990). Students' conceptual variations, as explained by Artigue (1990) and Azcarate (1995), are associated with some of the characteristics of function rather than with the concept on its own, especially when visualized in the graphic representation.

Regarding the semiotic registers, there is an evident predominance of algebraic representations. Students basically follow the same procedure: they begin with an algebraic expression, contract it to a value chart of four values maximum for the independent variable, locate the ordered pairs in the Cartesian plane and then join the dots with a line (which may be straight or curve depending on the distribution of dots). However, in most of the cases, these students have many difficulties trying to transfer the function from its verbal, chart and/or graph form to the algebraic form, likewise, they struggle establishing a relation between the different systems of representation. These results match those of works by Lesh, Post and Behr (1987), Janvier (1987) and Cuestas (2005), which identify difficulties to verbally express proposed relations or to translate them from graphic form to algebraic form (Leinhardt, Zaslavsky y Stein, 1990), or from the chart and graph to its algebraic representation (Markovits, Bat-Sheva y Bruckheimer, 1986).

Conclusions

From the research point of view, the conclusions are as follows:

The students' notion of function is not that of a formal definition. Instead, students show a series of conceptual variations that, in some cases, resemble an intuitive notion; therefore, based on these results, an in-depth study of said conceptual variations is expected to be considered a topic for future research (at qualitative level), with the objective of identifying the level of appropriation driven by teachers' practices at elementary and high school academic institutions.

At semiotic registrations level, out of all available registrations, students prefer the algebraic representation of the topic of function. The idea or the need to translate the function that is initially expressed in tabular or graphic registrations to an expression seems unattractive. But the fact that they prefer the algebraic register is not an indication that they master it at all, since poor comprehension of the formal algebraic language was evidenced, this aspect hinders the student to conduct integration and construction exercises of different forms of representation of the concept of function. Proper knowledge of this variations of semiotic representation of the function would allow a flexibilization of the articulations among them and an improved understanding of the notion; so, this knowledge constitutes an investigative reference for the Mathematics and Statistics Department at UFPS to determine the characteristics of the teaching process developed by teachers in the differential calculus subject, aimed at providing the tools to guarantee a suitable appropriation of mathematical concepts to advance in diverse mathematical skills, which will inevitably improve the academic results and reduce indicators of repetition or desertion, thus, taking advantage of the scarce economic resources, since we are a high school public institution.

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PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

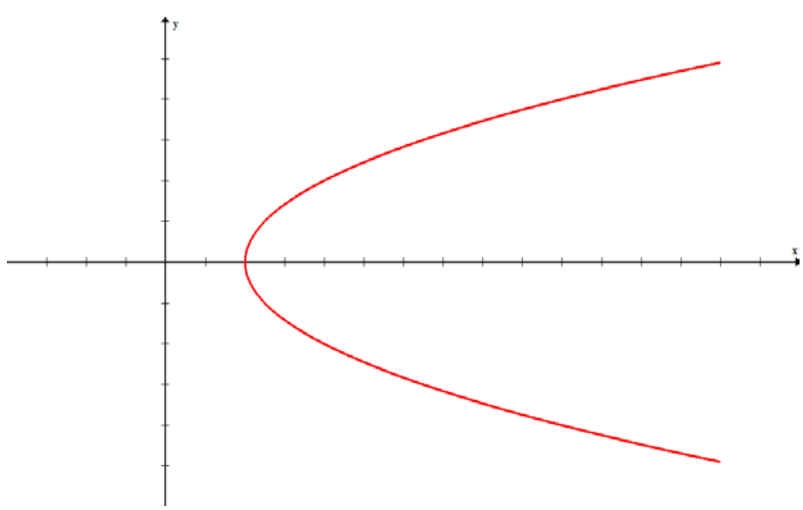
Panorama. Volumen XI. Número 20. Enero - junio 2017

Annex 1. Data-Collecting Instrument for the Pre-Test and the Post-Test

DEPARTMENT OF MATHEMATICS AND STATISTICS
“DIFFERENTIAL CALCULUS” DIAGNOSTIC

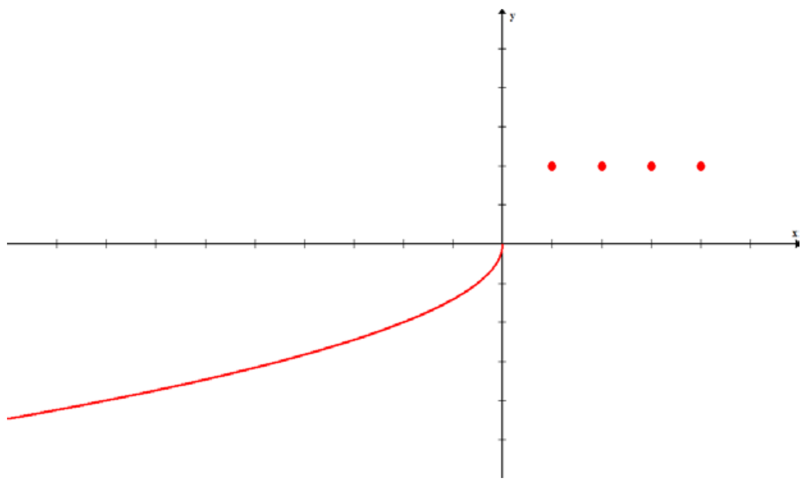
Instructions: the aim of this activity is to explore the knowledge of students enrolled in the differential calculus subject, regarding prior necessary concepts to successfully tackle the subject. It is not an exam; therefore, answers will not affect the subject’s grade. However, understanding the knowledge level of the students will allow an adaptation of the teacher’s work to the students’ reality. In that sense, it is recommended to complete it as honestly as possible, in order to guarantee an adequate planning of the classes by the teacher.

1. The following are a series of graphs in the Cartesian plane. Please identity which represent graphs of functions and justify your answer.

a.	Explanation:
	

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

b.	Explanation:
	

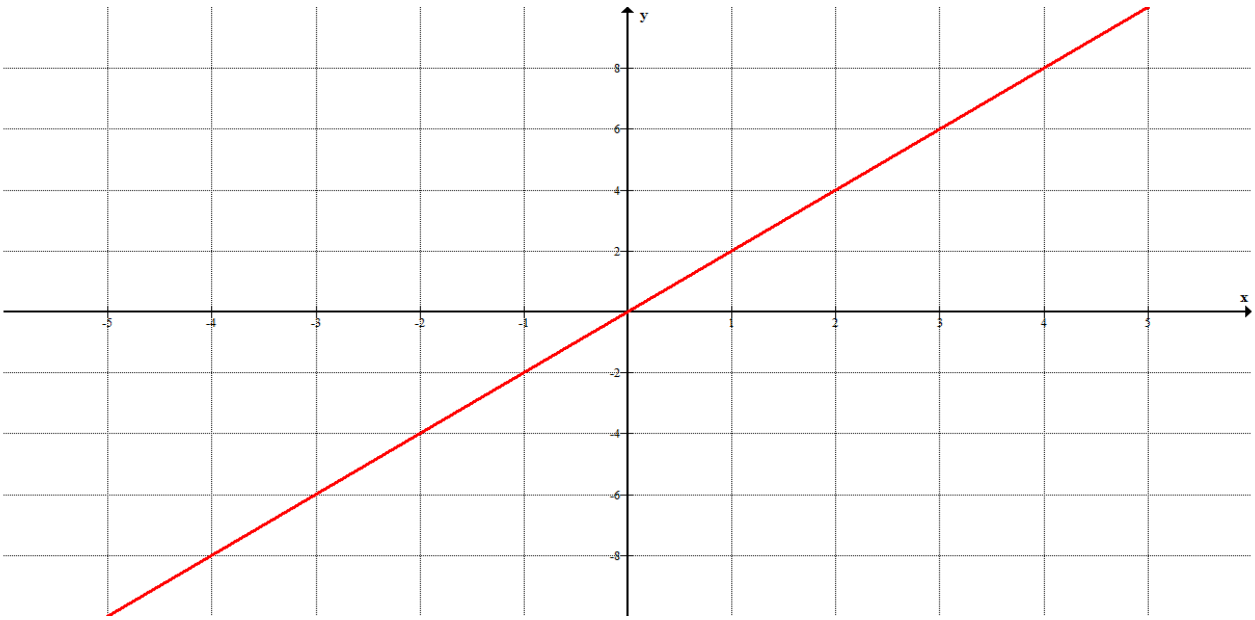
2. Read carefully and analyze the following formulation: Is there a function that assigns its square to each number that differs from zero? If so, one corresponds to zero?

Yes ☐ No ☐ Explain.

Situation_1. Questions 3 and 4 are answered by taking the following into account: given the f function, in which $f\colon\{Real\ Numbers\} \rightarrow \{Real\ Numbers\}$, represented in this graph.

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017



3. From the following algebraic expressions, which corresponds to the graph?

a. $f(x) = x + 1$
b. $f(x) = -x$
c. $f(x) = {}^{x}/_{2}$
d. $f(x) = 2x$

4. Which of the following ordered pairs is constituted bye elements of the graph of the f function?

$$\{(0,1),(1,2),\left(\frac{1}{2},1\right),(-2,-4),(4,8),(100,200)\}$$

Justify your answer.

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

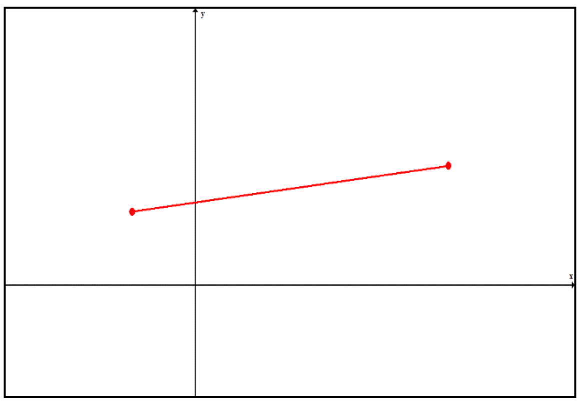
Panorama. Volumen XI. Número 20. Enero - junio 2017

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

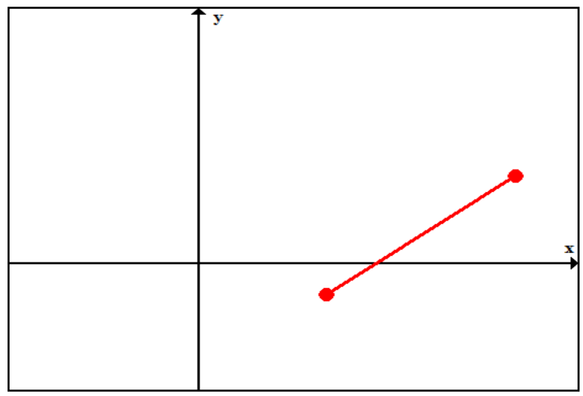
Panorama. Volumen XI. Número 20. Enero - junio 2017

5. Identify which of the following graphs represents a function with a domain of $\{x/2 \leq x \leq 6\}$ and a range of $\{f(x)/-1 \leq f(x) \leq 4\}$

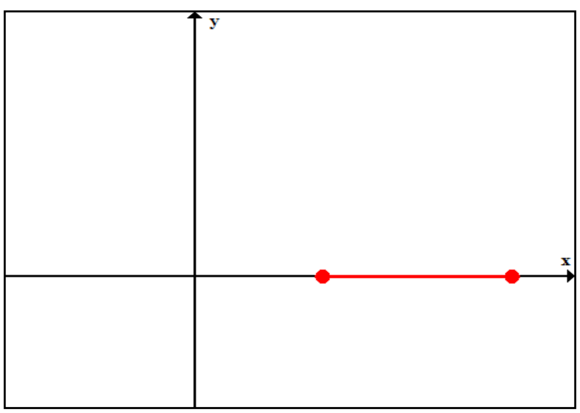
a.



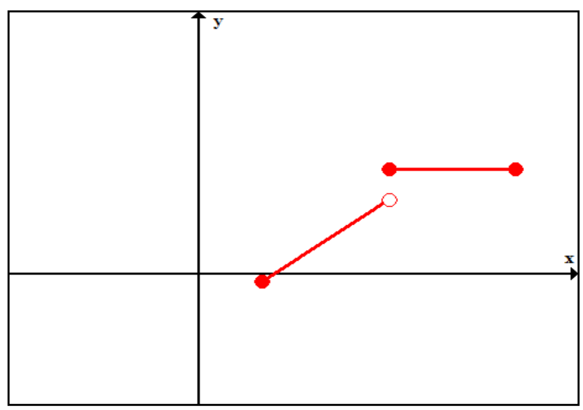
b.



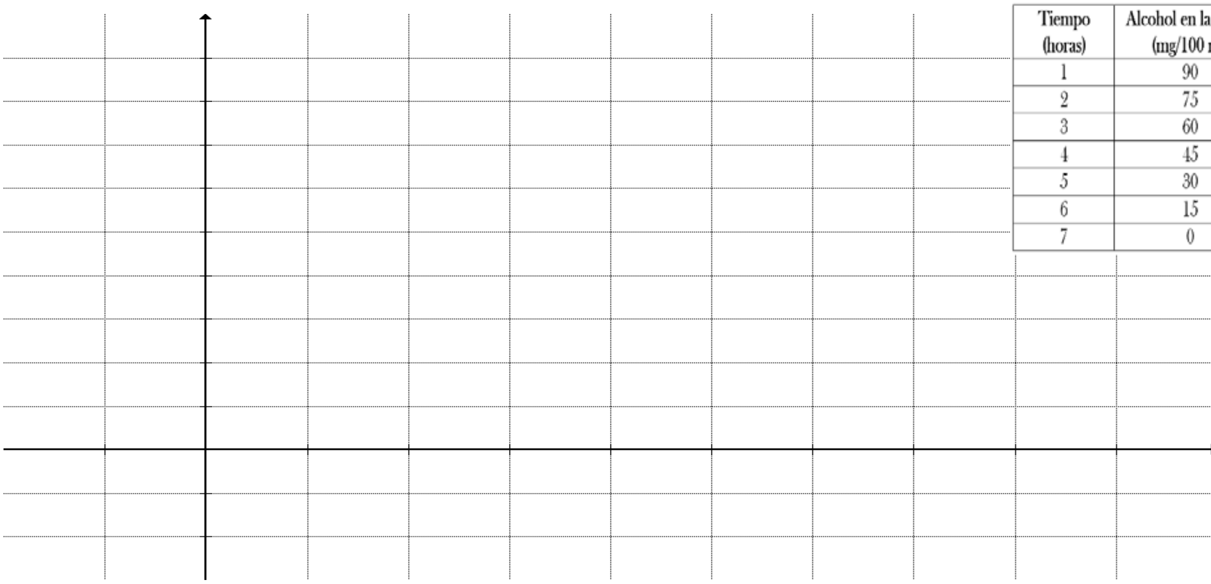
c.



d.



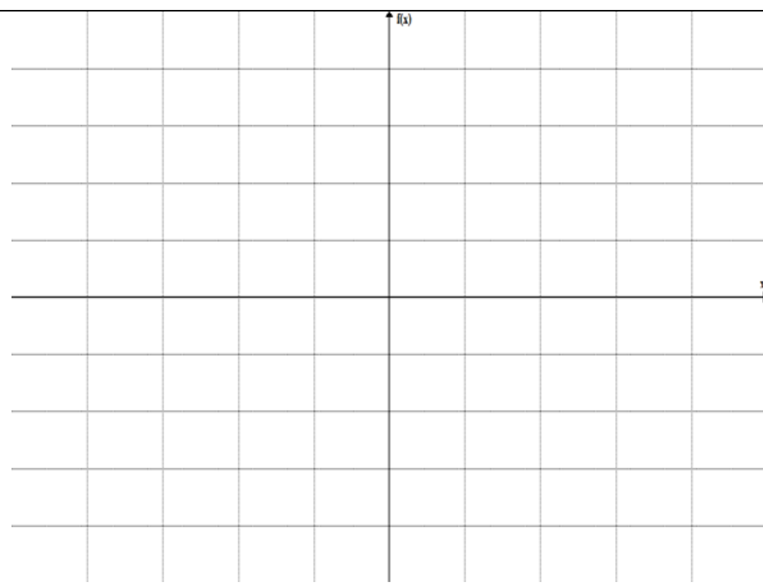
6. Graphically represent the information provided in the chart to answer the next question: what is the alcohol content in blood after drinking five beers?



Tiempo (horas)	Alcohol en la sangre (mg/100 ml)
1	90
2	75
3	60
4	45
5	30
6	15
7	0

7. Draw the graph of the following function:

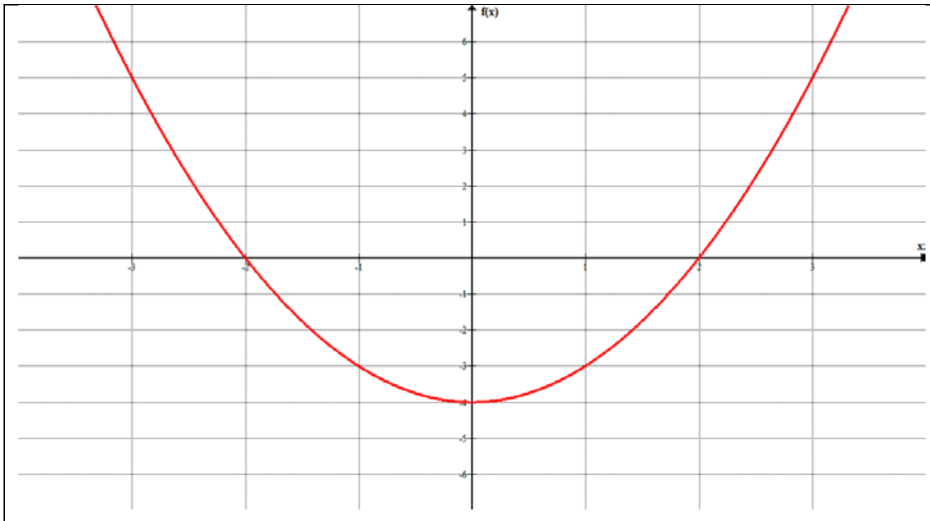
$f(x) = \frac{1}{x}$ for each $x \in R - \{0\}$



9. According to the next graph, what is the algebraic expression for $f(x)$ in terms of x?

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

	$f(x) =$ Justification:

10. For each of the given situations, propose an algebraic expression to represent it. Also, defend if it represents an example of a function.

a. The length of a rectangular lot of land is three times its width. Find the expression that defines the area in function of the lot's width.

b. A rectangle has an area of $12m^2$. Find the expression that expresses its perimeter *P* in terms of the length of one of its sides.

PANORAMA ISSN DIGITAL: 2145-308X ISSN: 1909-7433

Panorama. Volumen XI. Número 20. Enero - junio 2017

STUDENT INFORMATION

Age (as of today): ________________ Gender: Female □ Male □

Socio-economic level: _____________ Year of high school graduation: ____________

Type of educational institution from which you graduated: Public □ Private □

Your degree is: Academic □ Technical □ Validation □ Other □

Score obtained in *Pruebas Saber 11*: __________________________